ECE 3040 Microelectronic Circuits

Exam 1

February 5, 2004

Dr. W. Alan Doolittle

Print your name clearly and largely: Solutions

Instructions:
Read all the problems carefully and thoroughly before you begin working. You are allowed to use 1 new sheet of notes (1 page front and back) as well as a calculator. There are 100 total points. Observe the point value of each problem and allocate your time accordingly. SHOW ALL WORK AND CIRCLE YOUR FINAL ANSWER WITH THE PROPER UNITS INDICATED. Write legibly. If I cannot read it, it will be considered a wrong answer. Do all work on the paper provided. Turn in all scratch paper, even if it did not lead to an answer. Report any and all ethics violations to the instructor. Good luck!

Sign your name on ONE of the two following cases:

I DID NOT observe any ethical violations during this exam:

I observed an ethical violation during this exam:
Class Totals

Number of Tests = 29
Point Value of Problem =
Individual Problem Average = 90.9
Exam Average = 56.93
Exam Standard Deviation = 13.58
Exam Max = 86
Exam Min = 30

Section D
Class Totals

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Exam Average = 59.97

Exam Standard Deviation = 11.67

Exam Max = 82

Exam Min = 37
First 25%  Multiple Choice and True/False (Circle the letter of the most correct answer)

1.) (2-points) True or False: An intrinsic material has equal concentrations of electrons and holes.
2.) (2-points) True or False: A semiconductor material has a wider bandgap than an insulator.
3.) (2-points) True or False: An intrinsic semiconductor is also a degenerate semiconductor.
4.) (2-points) True or False: Direct bandgap semiconductors are used as light emitters (LEDs and LASERS) because of their efficient conversion of electrons to photons.
5.) (2-points) True or False: A zincblende crystal structure has more atoms in the unit cell than does the diamond crystal structure.

Select the best answer or answers for 6-10:

6.) (3-points) A “new” semiconductor consists of fictitious group II elements Dr, Do, Is and group VI elements Go and Od in equal composition. To within 1%, what is the correct reduced semiconductor notation for this remarkable compound?
   a.) DrDolsGoOd
   b.) Dr0.333Do0.333Is0.333Go0.5Od0.5
   c.) Dr0.26Do0.26Is0.26Go0.26Od0.26
   d.) Cannot be determined from the information given.
   e.) You cannot have more than 4 elements making up a semiconductor.

7.) (3-points) Which of the following mathematical expressions describes the free electron concentration? Hint: G_C is the density of states function, and f(E) is the fermi distribution function
   a.) \( n = G_C f(E) \)
   b.) \( n = G_C \frac{df(E)}{dE} \)
   c.) \( n = \sum G_C f(E) dE \)
   d.) \( n = G_C \int f(E) dE \)
   e.) Looks like I will be working as a Walmart clerk!

8.) (3-points) The following energy band diagram indicates the material is:
   a.) Non-degenerate n-type
   b.) Degenerate n-type
   c.) intrinsic
   d.) Non-degenerate p-type
   e.) Degenerate p-type

9.) (3-points) For the electron and hole shown in the following band diagram circle all that are true.
   a.) The electron experiences an electric field that will accelerate it.
   b.) The hole experiences an electric field that will accelerate it.
   c.) The hole will tend to stay where it currently is.
   d.) The device is in a transient state
   e.) The device has to be in equilibrium
10.) (3-points) Effective mass...
   a.) ... of an electron is always smaller than for a hole.
   b.) ... takes into account many different forces acting on the carrier that would otherwise not be present in free space
   c.) ... is always less than the mass of an electron in vacuum
   d.) ... determines (in part) how easy it is to move a carrier through the material
   e.) ... is a fundamental constant that does not change from material to material.

Second 25% Short Answer ("Plug and Chug"):
For the following problems (11-12) use the following material parameters:

\[ n_i = 1 \times 10^{-14} \text{ cm}^{-3}, \quad N_D = 8 \times 10^{17} \text{ cm}^{-3} \text{ donors} \quad N_A = 6 \times 10^{12} \text{ cm}^{-3} \text{ acceptors} \]
\[ m_n^* = 0.2 m_0, \quad m_p^* = 1.2 m_0 \]
Electron mobility, \( \mu_n = 900 \text{ cm}^2/\text{Vsec} \) Hole mobility, \( \mu_p = 10 \text{ cm}^2/\text{Vsec} \)

**Temperature = 150 degrees C**

11.) (10-points) Assuming total ionization, what is the electron and hole concentrations and is the material p or n-type?

\[ N_D > N_A \]
\[ n = N_D = 8 \times 10^{17} \text{ cm}^{-3} \]
\[ \rho = \frac{n_i^2}{n} = 1.25 \times 10^{-46} \text{ cm}^{-3} \]

12.) (15-points) What is the bandgap of the semiconductor material?

\[ N_V = 2 \left( \frac{m_p^* (kT)}{2 \pi \hbar^2} \right)^{3/2} \]

Similarly
\[ = 5.5 \times 10^{25} \text{ cm}^{-3} \]

\[ N_c = 2 \left[ \frac{m_n^* (kT)}{2 \pi \hbar^2} \right]^{3/2} = \left( \frac{0.2 (9,11 \times 10^{-31}) (8.617 \times 10^{-5}) (1.6 \times 10^{-19}) (150 + 273)}{2 \pi \hbar^2 (6.63 \times 10^{-34}) (1.17 \times 10^{-34})} \right)^{3/2} \]

\[ = 3.74 \times 10^{24} \text{ cm}^{-3} \]

\[ E_g = 2 kT \ln \left( \frac{n_i}{\sqrt{N_c N_V}} \right) = -2 (8.617 \times 10^{-5}) (150 + 273) \ln \left[ \frac{1 \times 10^{-14}}{1.5 \times 10^{-24} (2800)} \right] \]

\[ E_g = 6.57 \text{ eV} \]
**Third 25% Problems (3rd 25%)**

13.) (25-points total)

A 500 μm thick semiconductor at room temperature (27 degrees C) has the following parameters:
- Electron mobility, $\mu_e = 500 \text{ cm}^2/\text{V} \cdot \text{Sec}$
- Substrate intrinsic concentration, $n_i = 1 \times 10^{10} \text{ cm}^{-3}$
- The bandgap, $E_g = 1.1 \text{ eV}$

The substrate doping is linearly graded such that $N_d = 100n_i + 1 \times 10^6 x$, where $x$ is in cm.

**Sketch**

$N_0 \gg n_i$

$$E_F - E_i = AT \ln \left( \frac{100n_i + 1 \times 10^6 x}{n_i} \right)$$

$0.11 \text{ ev}$

$E_F$

$E_i$

$b.)$ (18 points) Determine an expression for the electric field verses position in the material:

$$J_n = 0 = q \mu_n \frac{\partial n}{\partial x} + q V_n \nabla n$$

$$0 = n \frac{\partial}{\partial x} + \frac{hT}{q} \nabla n$$

$$\frac{\partial E}{\partial x} = - \frac{hT}{q} \frac{\partial}{\partial x} \left( \frac{100n_i + 1 \times 10^6 x}{100n_i + 1 \times 10^6 x} \right)$$

$$\frac{\partial E}{\partial x} = - \frac{hT}{q} \left( \frac{1 \times 10^6 \text{ cm}^{-4}}{100n_i + 1 \times 10^6 x} \right) \frac{1 \times 10^6 \text{ cm}^{-4}}{100n_i + 1 \times 10^6 x}$$

$$E = - \frac{2.59}{1 + 1 \times 10^4 x} \frac{V}{\text{cm}}$$
Alternative Solution

\[ E_f - E_i = \frac{h}{2T} \ln \left( \frac{n_i}{n_i} \right) \]

\[ E_i = E_f - \frac{h}{2T} \ln \left( \frac{100n_i + 1e16 x}{n_i} \right) \]

\[ \varepsilon = \frac{1}{q} \frac{dE_i}{dx} = \frac{dE_f}{dx} - \frac{h}{q} \left( \frac{1e16}{100n_i + 1e16 x} \right) \]

\[ \varepsilon = -0.0259 [V] \left( \frac{1e16}{100n_i + 1e16 x} \right) \]

\[ \varepsilon = \frac{-259}{1 + 1e4 x} \text{ V/cm} \]
When the dinosaurs walked the planet, a light was turned onto a 10 μm thick InP semiconductor held at room temperature (27 degrees C). InP has a problem that the surfaces are really efficient recombination centers. Thus, the excess electron concentration at the surfaces will always remain 0 cm⁻³. The light uniformly generates $10^{18}$ additional holes per cm⁻³ per second. Determine the excess electron concentration in the InP for all positions AND plot your result, labeling the graph. Assume a minority carrier diffusion length of 0.1 μm and the mobility at room temperature is 3000 cm²/V·sec.

Given: $0 = D_n \frac{d^2 \Delta n_p}{dx^2} - \frac{\Delta n_p}{\tau_n}$
General Solution is: $\Delta n_p(x) = Ae^{-\frac{x}{\tau_n}} + Be^{\frac{x}{\tau_n}}$

Given: $0 = D_n \frac{d^2 \Delta n_p}{dx^2} - \frac{\Delta n_p}{\tau_n} + G_L$
General Solution is: $\Delta n_p(x) = Ae^{-\frac{x}{\tau_n}} + Be^{\frac{x}{\tau_n}} + G_L \tau_n$

Given: $0 = D_n \frac{d^2 \Delta n_p}{dx^2}$
General Solution is: $\Delta n_p(x) = A + Bx$

Given: $0 = D_n \frac{d^2 \Delta n_p}{dx^2} + G_L$
General Solution is: $\Delta n_p(x) = Ax^2 + Bx + C$

Given: $0 = D_n \frac{d^2 \Delta n_p}{dx^2} + G_{LO} f(x)$
General Solution is: $\Delta n_p(x) = \left[-\frac{G_{LO}}{D_n} \int f(x) dx \right] + Bx + C$

Given: $\frac{d\Delta n_p}{dt} = -\frac{\Delta n_p}{\tau_n}$
General Solution is: $\Delta n_p(t) = \Delta n_p(t=0) e^{-\frac{t}{\tau_n}}$

Sedative
Silver is on
Light is on

\[ \frac{\Delta n_p(x)}{\tau_n} = 0_n \int_0^x \frac{\Delta n_p}{\tau_n} dx + G_L \]

\[ \frac{\Delta n_p}{\tau_n} = \sqrt{D_n \tau_n} \]

0.1 μm = 0.1 e⁻⁴ cm

\[ \tau_n = \frac{\sqrt{\mu_n \frac{e^4}{q}}} {D_n} \]

\[ \tau_n = \frac{(0.1 e⁻⁴)^2}{0.0259 \approx (1000 cm²/V·sec)} \]

\[ \tau_n = 1.287 \rho s \]

\[ G_L \tau_n = 1.287 e^6 \text{ cm}^{-3} \]
B.C. \[ \Delta n_p(x=0) \Rightarrow A + B + G_L \Delta n = 0 \]

\[ \downarrow \]

\[ \beta = -(G_L \Delta n + A) \]

\[ \Delta n_p(x = 10 \mu m = 0.001 cm) = A e^{-100} + B e^{+100} + G_L \Delta n = 0 \]

Here we see for a reasonable result \( \Delta n_p \gg 0 \), \( B \) will be small!

\[ e^{-100} A = -(G_L \Delta n) - (\beta e^{100}) \]

\[ = -(G_L \Delta n) + (G_L \Delta n + A)(e^{100}) \]

\[ = -G_L \Delta n + G_L \Delta n e^{100} + A e^{100} \]

\[ A (e^{-100} - e^{100}) = G_L \Delta n (-1 + e^{100}) \]

\[ A = \frac{G_L \Delta n (-1 + e^{100})}{e^{-100} + e^{100}} \]

From \# above:

\[ B = -G_L \Delta n - \frac{G_L \Delta n (-1 + e^{100})}{e^{-100} + e^{100}} \]

\[ = -G_L \Delta n \left( \frac{-1 + e^{100}}{e^{-100} + e^{100}} + 1 \right) \quad \text{(Note:} \approx 2.6 G_L \Delta n) \]

\[ \therefore \Delta n_p(x) = G_L \Delta n \left[ 1 + \left( \frac{1 + e^{100}}{e^{-100} + e^{100}} \right) \right] - \frac{1 + e^{100}}{e^{-100} + e^{100}} + \frac{1}{e^{-100} + e^{100}} + \frac{x}{e^{-100} + e^{100}} \]

\[ \Delta n_p(x) = 1.287 e^6 \left[ 1 + \left( \frac{1 + e^{100}}{e^{-100} + e^{100}} \right) \right] - \frac{1 + e^{100}}{e^{-100} + e^{100}} + \frac{1}{e^{-100} + e^{100}} + \frac{x}{e^{-100} + e^{100}} \]

Note: As will be done for the BJT later, this can be reduced to \( \text{cm}^{-3} \)