ECE 3040 Microelectronic Circuits

Exam 1

February 21, 2011

Dr. W. Alan Doolittle

Print your name clearly and largely: Solutions

Instructions:
Read all the problems carefully and thoroughly before you begin working. You are allowed to use 1 new sheet of notes (1 page front and back) as well as a calculator. There are 100 total points. Observe the point value of each problem and allocate your time accordingly. SHOW ALL WORK AND CIRCLE YOUR FINAL ANSWER WITH THE PROPER UNITS INDICATED. Write legibly. If I cannot read it, it will be considered a wrong answer. Do all work on the paper provided. Turn in all scratch paper, even if it did not lead to an answer. Report any and all ethics violations to the instructor. Good luck!

Sign your name on **ONE** of the two following cases:

I DID NOT observe any ethical violations during this exam:

I observed an ethical violation during this exam:

First 33% Multiple Choice and True/False
(Circle the letter of the most correct *answer or answers*)

1.) (3-points) True or [✓ False:] The stronger the chemical bond, the smaller the energy bandgap.
2.) (3-points) True or [✓ False:] The Pauli-Exclusion principle says that no two electrons, regardless of spin, can have the same energy at the same location in space.
3.) (3-points) [✓ True] or False: If the Fermi-energy were located above the conduction band, there would be many occupied states in the conduction band despite the fact the conduction band states are normally considered to be mostly empty.
4.) (3-points) [✓ True] or False: The intrinsic concentration of electrons/holes results from the extremely unlikely event of multiple phonons (lattice vibrations) adding up to enough energy to break a valence electron off the atom.
5.) (3-points) True or [✓ False:] As doping is added to the semiconductor the average fermi-energy remains constant.
6.) (3-points) True or [✓ False:] The density of states (not the fermi-distribution function) predicts that at higher energies fewer electrons are found.
7.) (3-points) True or [✓ False:] Auger Recombination occurs mostly at low electron concentrations where electrons have the "room" to move around.

Select the **best** answer or answer**s** for 6-10:

8.) (4-points) The minority carrier diffusion equation ...
   a.) ... can predict drift currents .
   [✓ b.)] ... cannot be used when drift current is present
   c.) ... can only determine the majority carrier current.
   [✓ d.)] ... is a simplification of the current continuity equation.
   e.) ... is something I really do not understand and thus I might not pass this exam!

9.) (4-points) The probability of occupying a state at energy $E=E_f$ (where E_f is the fermi-energy) is...
   a.) ...essentially 1.
   b.) ...essentially 0.
   [✓ c.)] ... equals 0.5.
   d.) ... not known without knowing the density of states at that energy.

10.)(4-points) The appropriate equation to use for a n-type degenerate semiconductor to determine the electron concentration is:

a) $n = N_c e^{(E_f - E_c)/kT}$

b) $n = n_i e^{(E_f - E_i)/kT}$

[✓ c)] $n = N_c \frac{2}{\sqrt{\pi}} F_{1/2}(\eta_c)$

d) $n = \frac{N^+_D - N^-_A}{2} + \sqrt{\left(\frac{N^+_D - N^-_A}{2}\right)^2 + n_i^2}$

Second 17% Short Answer ("Plug and Chug"):
For the following problems (11-12) use the following material parameters and <u>assuming total ionization</u>:

Germanium:
n_i=1e14 cm^{-3} (not a mistake)　　N_A=1e15 cm^{-3} acceptors　　m_p^*=0.36m_o　　m_n^*=0.55m_o
E_G=0.66 eV　　Electron mobility, μ_n = 1800 cm^2/Vsec　　Hole mobility , μ_p = 150 cm^2/Vsec
Temperature=27 degrees C

11.)(10-points) Where is the fermi energy (relative to the valence band which is referenced to zero energy)?

Method 1

$$E_i = \frac{E_g}{2} + \frac{3kT}{4}\ln\left(\frac{m_p^*}{m_n^*}\right)$$

$$= \frac{0.66}{2} - 0.00823$$

$$E_i = 0.321\,eV$$

$$E_f - E_i = -kT\ln\left(\frac{10^{15}}{10^{14}}\right)$$

$$= -0.0596\,eV$$

$$E_f = -0.059 + 0.321$$

$$\boxed{E_f = 0.261\,eV}$$

note:

$$p = \frac{N_A - N_D}{2} + \sqrt{\left(\frac{N_A - N_D}{2}\right)^2 + n_i^2}$$

$$= 1.01\times10^{15}\,cm^{-3} \sim N_A$$

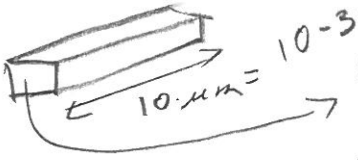
Method 2

$$p = N_V e^{(E_V - E_f)/kT}$$

$$N_V = 2.51\times10^{19}(0.36)^{3/2} = 5.42\times10^{18}\,cm^{-3}$$

$$E_V - E_f = -0.22\,eV \Rightarrow E_f = 0.22\,eV$$

12.) (7-points) What is the resistance of the semiconductor if it has 1 um^2 area and 10 um length?



$10\,\mu m = 10^{-3}\,cm$

Area = $1\,\mu m^2$
$= 10^{-8}\,cm^2$

$$\rho = \frac{1}{q(\mu_n n + \mu_p p)}$$

$$R = \rho\frac{L}{A} = \rho\frac{10^{-3}\,cm}{10^{-8}\,cm^2}$$

$$n_i^2 = np$$

$$(10^{14})^2 = n\,10^{15}$$

$$n = 1e13\,cm$$

$$\rho = \frac{1}{(1.6e{-19})(1800(1e13) + 150(1e15))}$$

$$= 37.2\,\Omega\text{-}cm$$

$$\boxed{R = \frac{37.2\,\Omega\text{-}cm}{10^{-5}} = 3.7\ \text{Megaohms}}$$

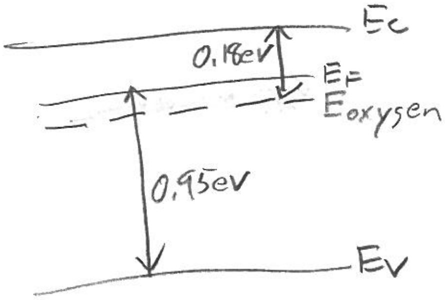
13.) (20-points total) Defects creating traps:

Oxygen makes a donor trap state 0.18 eV below the conduction band, E_C, edge in silicon. If the silicon is doped **n-type** and has a **hole** concentration of 2130 cm^{-3}, what is the concentration of *occupied* oxygen trap states? Assume the concentration of oxygen, [O], is small enough not to affect the overall doping and is equal to [O]=1e14 cm^{-3}, the energy bandgap is 1.12 eV and the effective density of states for electrons and holes, N_C = is 2.83e19 cm^{-3} and N_V is 1.82e19 $cm^{-3}$for silicon.

Hint: Since the oxygen defect state energy is referenced from the conduction band, E_C, it may be helpful to re-write the energy terms in the fermi-distribution function referenced from the conduction band energy.

Most direct approach

$$p = N_V e^{(E_V - E_F)/kT}$$



$$2130 = 1.82e19\, e^{-E_F/kT}$$

$$E_F = 0.9483\ eV$$

$$f(E) = \frac{1}{1+e^{[\underbrace{(1.12-0.18)}_{E = \text{Energy}} - \underbrace{0.9483}_{E_F}]/0.0259}}$$

$$= 0.5796$$

$$[O^-] = f(E)[O]$$

$$= 0.5796\,(1e14)\ cm^{-3}$$

$$\boxed{[O^-] = 5.79e13\ cm^{-3}}$$

alternatively (more convoluted approach)
see next page

13.) (20-points total) Defects creating traps:

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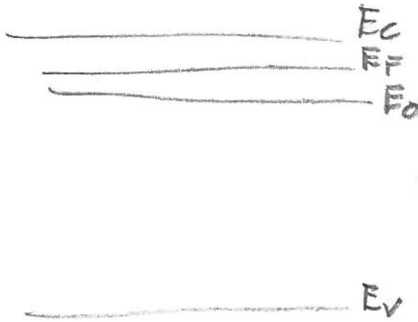


$$n_i = \sqrt{N_c N_v}\, e^{-E_g/2kT} \quad kT \rightarrow 0.0259$$

$$n_i = 9.23\,e9\ cm^{-3}$$

$$np = n_i^2$$

$$n = \frac{(9.23e9)^2}{2130} = 4\,e\,16\ cm^{-3}$$

$$n = N_c\, e^{-(E_c - E_F)/kT}$$

$$4e16 = 2.82e19\, e^{-(E_c - E_f)/0.0259}$$

$$E_c - E_f = 0.1698\ eV$$

$$f(\underset{\substack{\uparrow \\ 0.18eV\ below\ E_c}}{E}) = \frac{1}{1+e^{(E-E_f)/kT}} = \frac{1}{1+e^{\frac{(E_c - E_f) - (E_c - E)}{kT}}}$$

$$\underset{0.169}{\uparrow}\ (E_c - E_f) \qquad \underset{0.18}{\uparrow}\ (E_c - E)$$

$$f(E) = \frac{1}{1+e^{-0.01/0.0259}}$$

$$= 0.59\bar{6}$$

$$[O]^- = 1e14\,(0.59\bar{6})\ cm^{-3}$$

$$\boxed{[O]^- = 5.96\,e13\ cm^{-3}}$$

13.) (20-points total) Defects creating traps:

Oxygen makes a donor trap state 0.18 eV below the conduction band, E_C, edge in silicon. If the silicon is doped **n-type** and has a **hole** concentration of 2130 cm^{-3}, what is the concentration of *occupied* oxygen trap states? Assume the concentration of oxygen, [O], is small enough not to affect the overall doping and is equal to [O]=1e14 cm^{-3}, the energy bandgap is 1.12 eV and the effective density of states for electrons and holes, N_C = is 2.83e19 cm^{-3} and N_V is 1.82e19 $cm^{-3}$for silicon.

Hint: Since the oxygen defect state energy is referenced from the conduction band, E_C, it may be helpful to re-write the energy terms in the fermi-distribution function referenced from the conduction band energy.

3rd approach

$$N_D^+ = \frac{N_D}{1+g_d\, e^{(E_f-E_D)/kT}}$$

$[O] = 1e14\ cm^{-3}$

$(1.12-0.18)\,eV$

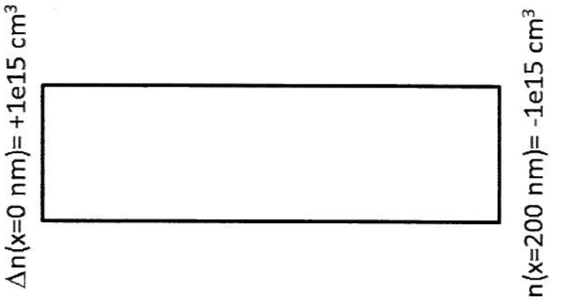
$$= \frac{1e14}{1+2e^{(0.9483-0.94)/0.0259}}$$

$$= 2.66e13\ cm^{-3} \Rightarrow \boxed{\text{occupied} = [O]-N_D^+ = 7.33e13\ cm^{-3}}$$

Differs from previous answer due to degeneracy factor

Pulling all the concepts together for a useful purpose:

14.) (30-points)



In a particular region of a transistor we will study in detail later, called the "base" region, there is a condition established where extra minority carriers are injected (added) at one side while the opposite side has extra minority carriers extracted (removed). If this region is 200 nm in length and the left end (x=0) has a 1e15 cm^{-3} more minority carriers than found in equilibrium while the right end (x=200 nanometers) has a 1e15 cm^{-3} fewer carriers than found in equilibrium, what is the minority carrier diffusion current density in the region. In this "base region", the minority carrier lifetime is 10 nanoseconds. The device is packaged inside an opaque plastic container so no light reaches the device and the device has been operating since Dr. Doolittle was born (ancient times ☺). Assume a minority carrier mobility of 4.0 cm^2/Vsec.

$G_L = 0$

$\frac{\partial \Delta n}{\partial t} = 0$

$$\boxed{\text{Given: } 0 = D_n \frac{d^2 \Delta n_p}{dx^2} - \frac{\Delta n_p}{\tau_n} \qquad \text{General Solution is: } \Delta n_p(x) = Ae^{-x/L_n} + Be^{+x/L_n}}$$

Given: $0 = D_n \frac{d^2 \Delta n_p}{dx^2} - \frac{\Delta n_p}{\tau_n} + G_L$ General Solution is: $\Delta n_p(x) = Ae^{-x/L_n} + Be^{+x/L_n} + G_L \tau_n$

Given: $0 = D_n \frac{d^2 \Delta n_p}{dx^2}$ General Solution is: $\Delta n_p(x) = A + Bx$

Given: $0 = D_n \frac{d^2 \Delta n_p}{dx^2} + G_L$ General Solution is: $\Delta n_p(x) = Ax^2 + Bx + C$

Given: $0 = D_n \frac{d^2 \Delta n_p}{dx^2} + G_{LO} f(x)$ General Solution is: $\Delta n_p(x) = \left[-\frac{G_{LO}}{D_N} \iint f(x)dx \right] + Bx + C$

get from Einstien Relationship

Given: $\frac{d\Delta n_p}{dt} = -\frac{\Delta n_p}{\tau_n}$ General Solution is: $\Delta n_p(t) = \Delta n_p(t=0)e^{-t/\tau_n}$

Given: $0 = -\frac{\Delta n_p}{\tau_n} + G_L$ General Solution is: $\Delta n_p = G_L \tau_n$

$L_n = \sqrt{D_n \tau}$

$= \sqrt{(0.0259)4(10e{-9})}$

$= 3.2e{-5}$ cm

$= 0.31$ um

200nm = 2e-5cm

Boundary Conditions:

$\Delta n(x=0) = 10^{15} \text{cm}^{-3} = A + B$

$\Delta n(x=200\text{nm}) = -10^{15} \text{cm}^{-3}$

$(10^{15} - B)e^{-(2e-5)/(3.2e-5)} + Be^{+(2e-5)/3.2e-5} = -10^{15} \text{cm}^{-3}$

$B(-0.535 + 1.868) = -1.535e^{15}$

$B = -1.151 \times 10^{15} \text{cm}^{-3} \Rightarrow A = 2.151 \times 10^{15} \text{cm}^{-3}$

Extra work can be done here, but clearly indicate which problem you are solving.

$$\therefore\ \Delta n(x) = 2{,}151\times10^{15}\, e^{-x/3{,}2e-5} - 1{,}151\times10^{15}\, e^{x/3{,}2e-5}$$

$$J_n = \underset{\downarrow}{q}\,\underset{\searrow}{[D_n]}\,\frac{d\,\Delta n}{dx}$$

$$= (1{,}6e-19)\,[(0{,}0259)4]\left[-\frac{2{,}151e15}{3.2e-5}e^{-x/3{,}2e-5} - \frac{1{,}151e15}{3{,}2e-5}e^{x/3{,}2e-5}\right]$$

$$= 5{,}18e-16\,[-2{,}151e15\ \ldots\ - 1{,}151e15,\ldots]$$

$$\boxed{J_n = -1{,}11\,e^{-x/3{,}2e-5} - 0{,}596\,e^{x/3{,}2e-5}\ A/cm^2}$$