Print your name clearly and largely: Solutions

Instructions:
Read all the problems carefully and thoroughly before you begin working. You are allowed to use 1 new sheet of notes (1 page front and back), your note sheet from the previous exams as well as a calculator. There are 105 total points in this exam (5 bonus points). The exam will be graded on a 100-point basis. Observe the point value of each problem and allocate your time accordingly. SHOW ALL WORK AND CIRCLE YOUR FINAL ANSWER WITH THE PROPER UNITS INDICATED. Write legibly. If I cannot read it, it will be considered a wrong answer. Do all work on the paper provided. Turn in all scratch paper, even if it did not lead to an answer. Report any and all ethics violations to the instructor. Good luck!

Sign your name on ONE of the two following cases:

I DID NOT observe any ethical violations during this exam:

I observed an ethical violation during this exam:
First 20% Multiple Choice and True/False (Select the most correct answer)

1.) (4-points) If the MOS capacitor shown in the diagram to the right were used in a MOSFET transistor, which of the following COULD result (more than one answer possible):
   a.) NMOS transistor biased into Cutoff mode
   b.) PMOS transistor biased into Cutoff mode
   c.) NMOS transistor biased into linear/triode mode
   d.) PMOS transistor biased into linear/triode mode
   e.) NMOS transistor biased into Saturation mode
   f.) PMOS transistor biased into Saturation mode

2.) (4-points) The amplifier below has which of the following filter shapes (circle the correct filter shape)?

3.) (4-points) In the MOSFET amplifier to the right, WITHOUT doing any calculations, estimate the voltage gain.
   a.) $g_m R_L$
   b.) $R_2 / (R_1 + R_2)$
   c.) One
   d.) Can not be determined without $V_T$
4.) (4-points) In the MOSFET transistor to the right, what is the voltage at the point in the channel indicated by the arrow?
   a.) $V_{DS} - V_T$
   b.) $V_{DS}$
   c.) $V_{GS}$
   d.) $V_{GS} - V_T$
   e.) Not enough information given to solve
   f.) 1 Volt (because it sounds like a nice round number)

5.) (4-points) Circle ALL the NMOS transistors below…

![NMOS transistors]
6.) (20-points) In the amplifier circuit below, Vin is constrained to be greater than 0V (Vin>0). Determine an expression relating Vout to Vin and determine the input impedance of the circuit. You may assume that the Op-Amp is ideal, the diode has a saturation current of 1e-14 A and the CVD model DOES NOT hold (i.e. use the full diode equation). **NOTE:** In this circuit, you cannot write a gain expression of the form Vout/Vin. You must write an expression relating Vout as a function of Vin (Vout=f(Vin)).

Bonus Points (must be completely correct or no points awarded): (5 points) Why must Vin be constrained to greater than 0V for this circuit to operate properly? This is a simplified example of a "compressor" Amplifier that regulates a signals magnitude by a logarithmic function.

\[ i_1 = i_2 = I_0 \left( e^{V_d/V_{th}} - 1 \right) \]

\[ V_L = V_T = 0 \text{V} \]

\[ V_{out+} = -V_d \]

\[ i_1 = \frac{V_{in}}{R_1} = I_0 \left( e^{V_d/V_{th}} - 1 \right) \]

\[ V_d = V_{th} \ln \left( \frac{\frac{V_{in}}{R_1}}{I_0} + 1 \right) \]

\[ V_{out+} = -0.0259 \ln \left( \frac{V_{th}}{1k(1e-14)} + 1 \right) \]

\[ V_{out+} = -0.0259 \ln \left( \frac{1k(1e-14)}{V_{th} + 1} \right) \text{V} \]

\[ R_{in} = R_1 = 1k\Omega \]
This problem was identical to the problem performed in class 3 days before the exam! See summer 2002 exam 3.

Pulling all the concepts together for a useful purpose:

7.) (60-points) I want to listen to an Atlanta Braves game on AM 750 radio operating at 750 KHz while rejecting AM 790's station at 790 KHz. In my lab, I build the following "tuned amplifier". Given the following circuit and material parameters, what is the AC voltage gain, \( V_{out}/V_{in} \) at 750KHz and also at 790 KHz (two separate answers)? You may assume capacitor, C1 (and only C1) has an infinite capacitance.

**Both Ld and Cd must be considered as finite, complex impedances. "If" your gain is a complex number, show it in magnitude phase form, i.e. Mag /Degrees form.** Additionally, consider the circuit to be operated at low frequencies where you can neglect all small signal capacitances. Grading will be based as such: 16 points for the plug and chug answers, 20 points for DC solution and 24 points for small signal analysis. **Hint:** Initially, perform all calculations using a combined impedance, \( Z_d \) (\( Z_d \) includes \( R_d, L_d, C_d \)), then substitute the proper value of \( Z_d \) at 750 and 790 KHz in at the end.

Gate Length, \( L = 2.5 \, \mu \text{m} \)

Gate Width, \( W = Z = 25 \, \mu \text{m} \)

Effective mobility, \( \mu_m = 200 \, \text{cm}^2/\text{VSec} \)

Oxide Thickness, \( t_{ox} = x_{ox} = 27.6 \, \text{nm} \)

Channel Length Modulation parameter, \( \lambda = 0.1 \, \text{V}^{-1} \)

Substrate Doping, \( N_A = 1.68 \times 10^{16} \, \text{cm}^{-3} \)

Oxide relative Dielectric Constant, \( \varepsilon_{\text{oxide}} = K_O = 3.9 \)

Substrate relative Dielectric Constant, \( \varepsilon_{\text{semiconductor}} = K_S = 11.7 \)

Substrate intrinsic concentration, \( n_i = 1 \times 10^{10} \, \text{cm}^{-3} \)

Dielectric Constant of free space, \( \varepsilon_0 = 8.854 \times 10^{-14} \, \text{F/cm} \)

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**Plug and Chug Answers** (As these answers are required to continue on with the problem, you may purchase these answers for 5 points each. Exam proctors should clearly mark values as "PURCHASED"):

\[ V_T = 1.25 \, \text{V} \]

\[ \phi_F = 0.37 \, \text{V} \]

\[ C_{ox} = 1.25 \times 10^{-7} \, \text{F/cm}^2 \]

\[ K_n = 250 \, \mu \text{A/V}^2 \]

\[ \phi_F = \frac{K_n}{V} \ln \left( \frac{1.68 \times 10^{16}}{1 \times 10^{10}} \right) = 0.37 \, \text{V} \]

\[ C_{ox} = 3.9 \left( \frac{8.854 \times 10^{-14}}{2.76 \times 10^{-6}} \right) = 1.25 \times 10^{-7} \, \text{F/cm}^2 \]

\[ K_n = \left( \frac{25}{2.5} \right) 200 (1.25 \times 10^{-7}) = 250 \, \mu \text{A/V}^2 \]

\[ V_T = 2 \left( 0.37 \right) + \frac{11.7 \left( 8.854 \times 10^{-14} \right)}{1.25 \times 10^{-7}} \sqrt{2 \left( 1.6 \times 10^{-19} \right) \left( 1.68 \times 10^{16} \right) (2) (0.37)} \]

\[ = 1.25 \, \text{V} \]
DC Analysis:

\[ \text{Id} = \frac{250 \text{e-}6}{2} \left( \frac{2.73 - 1.25}{1 + 0.1(10 - \text{Id} \cdot 1k)} \right) \]

\[ \text{Id} = \frac{2.738 \text{e-}4 \cdot 1 + 1 - 100 \text{Id}}{2} \]

\[ \text{Vo} = 5.476 \text{e-}4 - 1.02738 \text{Id} \]

\[ \text{Id} = 533 \mu A \]

Check:

\[ \text{Vo} = 10 - 10k \cdot (533 \text{e-}6) \]

\[ = 4.67 V > V_{GS} - V_T = (2.73 - 1.25) \]

\[ = 1.53 \]

Small Signal Parameters / Saturation Verified.

\[ g_m = \frac{\text{Id}}{V_{GS} - V_T} = \frac{533 \text{e-}6}{(2.73 - 1.25) / 2} = 720 \mu A/V \]

\[ r_o = \frac{1}{\frac{k_n}{2} (V_{GS} - V_T)^2} = 36.5 \text{ k } / \text{ } 2 \]
AC Solution

\[ V_{th} = \frac{R_{18}||R_{1}||R_{2}}{R_{18}||R_{1}||R_{2} + R_{16}} V_{in} \]

1) \[ V_{th} = 0.5 V_{in} \]
\[ V_{th} = R_{16} || R_{18} || R_{1} || R_{2} \]
\[ = 500 \Omega \]

2) \[ V_{SS} = V_{th} \]
3) \[ V_{out} = -(g_m V_{SS}) \left( \frac{R_{oll} || Z_D}{R_{oll}} \right) \]
\[ A_V = \left( \frac{V_{out}}{V_{SS}} \right) \left( \frac{V_{SS}}{V_{th}} \right) \left( \frac{V_{th}}{V_{in}} \right) \]
\[ = \frac{g_m (R_{oll} || Z_D)}{2} (1) (0.5) \]
\[ = -g_m \frac{R_{oll} || Z_D}{2} \]

What is \( Z_D \) @ 750 kHz?
Extra work can be done here, but clearly indicate with problem you are solving.

\[
\bar{Z}_d = \frac{L_d s}{C_d s} + R_d
\]

\[
\bar{Z}_d = \frac{L_d s}{C_d s + \frac{1}{s^2 L_c C_d}} + R_d
\]

\[
\bar{Z}_d = \frac{j w L_d}{1 + w^2 L_c C_d} + R_d
\]

Consider 3 cases

- \( f_1 = 750 \text{ kHz} \)
  \( \omega_1 = 2\pi f_1 = 4,712,388.98 \)
  \( Z_d = j (\omega_1) (100 e^{-6}) \)
  \( Z_d = 1k + j 0.1423 \omega_1 \)
  \( Z_d = 1k + j 670.751 Ohm \)
  \( \text{Roll } Z_d = 36,389 + j1980 \)
  \( \text{Av} = -360 e^{-6} \text{ (roll } Z_d) \)
  \( \text{Av} = -13.10 - j 0.7128 \text{ V/V} \)

- \( f_2 = 790 \text{ kHz} \)
  \( \omega_2 = 2\pi f_2 \)
  \( Z_d = \frac{j w L_d}{1 - \omega_2^2 (100 e^{-6}) (450.3 e^{-12})} \)
  \( Z_d = 1k + j \omega_2 0.000913 \)
  \( Z_d = 1k - j 4534.28 \text{ Ohm} \)
  \( \text{Roll } Z_d = 1485.25 - j 4233.77 \)
  \( \text{Av} = -360 e^{-6} \text{ (roll } Z_d) \)
  \( \text{Av} = -0.53 + j 1.524 \)

- \( w = \frac{2\pi f}{\sqrt{L_c C}} \) (at resonance)
  \( w^2 L_c C_d = 1 \)
  \( Z_d = \frac{j w L_d}{\omega} + R_d \)
  \( Z_d = 100 \)
  \( \text{Av} = -720 e^{-6} \text{ (36.5kOhm) } \)
  \( \text{Av} = -13.14 \text{ V/V} \)

\( \text{(Requires } f = \frac{1}{2\pi}\sqrt{\frac{C}{L}} \text{ of } f = 750,013.6 \text{ Hz) } \)  
  
I.E. Very sensitive 
Gain-freq. relationship
Extra work can be done here, but clearly indicate with problem you are solving.

This "tuned amplifier" is an example of a bandpass filter often used in Radio & TV circuits (greatly simplified version, of course). It has a frequency response that looks like this,

\[ \Delta \pi f_0 = \sqrt{\frac{1}{LC}} \]

In a "real implementation", \( R_d \) would be eliminated to increase the freq. selectivity.