Thevenin’s and Norton’s Equivalent Circuit Tutorial. (by Kim, Eung)

Thevenin’s Theorem states that we can replace entire network by an equivalent circuit that contains only an independent voltage source in series with an impedance (resistor) such that the current–voltage relationship at the load is unchanged.

Norton’s Theorem is identical to Thevenin’s Theorem except that the equivalent circuit is an independent current source in parallel with an impedance (resistor). Therefore, the Norton equivalent circuit is a source transformation of the Thevenin equivalent circuit.

They are Interchangeable
**How to find Thevenin's Equivalent Circuit?**

<table>
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| **Resistors and independent sources** | 1) Connect an open circuit between a and b.  
2) Find the voltage across the open circuit which is Voc.  
Voc = Vth.  
3) Deactivate the independent sources.  
Voltage source ➔ open circuit  
Current source ➔ short circuit  
4) Find Rth by circuit resistance reduction |
| **Resistors and dependent sources or independent sources** | 1) Connect an open circuit between a and b.  
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Voc = Vth.  
3) Connect a short circuit between a and b.  
4) Determine the current between a and b.  
5) Rth = Voc / Iab  
If there are only dependent sources.  
3) Connect 1 Ampere current source flowing from terminal b to a. It = 1 [A]  
4) Then Rth = Voc / It = Voc / 1 |

**Note:** When there are only dependent sources, the equivalent network is merely $R_{Th}$, that is, no current or voltage sources.
**How to find Norton's Equivalent Circuit?**

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| Resistors and independent sources | - Deactivate the independent sources.  
- Voltage source → open circuit  
- Current source → short circuit  
- Find Rt by circuit resistance reduction  
- Connect an short circuit between a and b.  
- Find the current across the short circuit which is \( I_{sc} \). |
| Resistors and dependent sources or Independent sources | 1) Connect a short circuit between a and b.  
2) Find the current across the short circuit which is \( I_{sc} \).  
\( I_{sc} = I_n \).  
If there are both dependent and independent sources.  
3) Connect a open circuit between a and b.  
4) Determine the voltage between a and b.  
\( V_{oc} = V_{ab} \)  
5) \( R_n = \frac{V_{oc}}{I_{sc}} \)  
If there are only dependent sources.  
3) Connect 1 Ampere current source flowing from terminal b to a.  
\( I_t = 1 \ [A] \)  
4) Then \( R_n = \frac{V_{oc}}{I_t} = \frac{V_{oc}}{1} \) |

**Note:** When there are only dependent sources, the equivalent network is merely \( R_{Th} \), that is, no current or voltage sources.

**References**

\* Voltage Divider.

When you have multiple resistors in a single loop, the source voltage will be divided according to KVL.

\[ V_S = V_1 + V_2 + V_3 \]

And since there exists only one loop, the current flowing through each resistor is the same as \( I \).

\[
\begin{align*}
V_1 &= I \cdot R_1 \\
V_2 &= I \cdot R_2 \\
V_3 &= I \cdot R_3
\end{align*}
\]

Therefore

\[ V_S = V_1 + V_2 + V_3 = I \cdot R_1 + I \cdot R_2 + I \cdot R_3 = I \left( R_1 + R_2 + R_3 \right) \]

\[ \therefore I = \frac{V_S}{R_1 + R_2 + R_3} \]

Thus, the voltage across the \( n \)th resistor \( R_n \) can be found as

\[ V_n = I \cdot R_n = \left( \frac{V_S}{R_1 + R_2 + R_3} \right) \cdot R_n \]

In general, we may repeat the voltage divider principle by

\[ V_n = \left( \frac{V_S}{\sum_{i=1}^{n} R_i} \right) \cdot R_n \]
**Example 1**

\[ V_{ab} = \frac{50}{3 + 1 + \frac{1}{4} \cdot (4114) + \frac{1}{5} \cdot (511515)} \times (4114) \]

We can separate the resistors into some groups of parallel-connected resistors.
*Current Divider.*

When we have multiple resistors in parallel connection, the source current will be divided into each parallel branch according to KCL

\[ I_S = I_1 + I_2 + I_3 \]

Since parallel-connected resistors can be simplified as one single resistor as \((R_1 \ || \ R_2 \ || \ R_3)\), the voltage across each resistor is the same as \(V\).

\[
\begin{align*}
I_1 &= \frac{V}{R_1} \\
I_2 &= \frac{V}{R_2} \\
I_3 &= \frac{V}{R_3}
\end{align*}
\]

Therefore

\[
I_S = I_1 + I_2 + I_3 = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}
\]

\[
= V \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)
\]

\[
\therefore V = \frac{I_S}{\left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)}
\]

Thus, the current flowing through nth resistor \(R_n\) can be found as

\[
I_n = \frac{V}{R_n} = \frac{I_S}{\left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)} \cdot \frac{1}{R_n}
\]

In general, we can repeat the current divider principle by

\[
I_n = \left( \sum_{i=1}^{n} G_n \right) \cdot G_n \quad (G \text{ is conductance})
\]

\[
G_n = \frac{1}{R_n}
\]
example 1)

\[ I_3 = \frac{10}{\left(\frac{1}{4} + \frac{1}{4}\right)} = \frac{10}{\frac{1}{4} + \frac{1}{4}} \]

\[ \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \]

\[ I_3 = \frac{10}{\frac{1}{2}} = 20 \]

\[ R_1 = 2 \Omega \]
\[ R_2 = 2 \Omega \]
\[ R_3 = 4 \Omega \]
\[ R_4 = 4 \Omega \]

\[ I_3 \] is series-connected to the current source, therefore the current flowing across \( R_1 \) and \( R_2 \) is just the same as 10 A. However, \( R_3 \) and \( R_4 \) are parallel-connected, so the current will be divided into two branches.
* Thevenin's and Norton's Equivalent Circuits

**Example 1**

\[ \text{50V} \]
\[ \begin{array}{c}
5\Omega \\
20\Omega \\
\end{array} \]
\[ \begin{array}{c}
a \\
b \\
\end{array} \]

**Solution**

1. **Deactivate Voltage Source**
   (Voltage source \( \Rightarrow \) Short Circuit)

   \[ \begin{array}{c}
5\Omega \\
20\Omega \\
\end{array} \]

2. Simplify the circuit and find \( R_{th} \)

   \[ \begin{array}{c}
5\Omega \\
20\Omega \\
\end{array} \]

   \[ R_{th} = (5||20) + 4 = 8\Omega \]

3. **Find Open-Circuit Voltage across ab**

   \[ V_{oc} = V_{ab} = V_{th} \]

   \[ V_x \quad \rightarrow \quad i_x \]

   \[ V_x = V_a \quad \text{i.e. node a is open} \]

   \[ \text{no voltage drop across 4\Omega resistor} \]

   \[ V_{ab} = V_{th} = 50 \times \frac{20}{20+5} = 40\text{V} \]

**Thevenin's Circuit is**

\[ \begin{array}{c}
8\Omega \\
\end{array} \]
\[ \begin{array}{c}
a \\
b \\
\end{array} \]

\[ \text{40V} \]
This Thévenin's Circuit can be converted to Norton's Circuit as:

\[ V_{\text{th}} = 40 \text{V} \]
\[ R_{\text{th}} = 8 \Omega \]

\[ I_n = \frac{V_{\text{th}}}{R_{\text{th}}} = \frac{40}{8} = 5 \text{A} \]

\[ \Rightarrow \]

\[ V_{\text{in}} = 5 \text{A} \]
\[ 8 \Omega = R_{\text{n}} \]

Note: When you convert Thévenin's Circuit to Norton's Circuit, the direction of current flow of the current source in Norton's Circuit should be matched with the voltage source in Thévenin's Circuit.

Example 2):

1. \textit{Deactivate independent sources}:
   - \textit{Voltage source} → short
   - \textit{Current source} → open

2. \textit{Simplify the circuit and find} \( R_{\text{th}} \):

   \( R_{\text{th}} = (10/140) + 4 = 12 \Omega \)

3. \textit{Find open-circuit voltage across ab}:
   - \( i_x = 0 \) since node a is open.
   - No voltage drop across 4-Ω resistor.
   - We can ignore 4-Ω resistor.
If we set the minus terminal of the voltage source as ground, then the voltage at node-\(x\) is \(V_{ab}\).

Apply KCL to node-\(x\):

\[
\frac{V_{ab}}{10} + \frac{V_{ab}}{40} + 2 = 0
\]

\[
V_{ab} = -8 \, \text{V} = V_{th}.
\]

Therefore, Thevenin's Circuit is:

\[
V_{th} = 8 \, \text{V}
\]

(The polarity of the voltage source is reversed since Thevenin voltage source is minus value.)

As you know, the impedance in Thevenin's Circuit is the same as the impedance in Norton's Circuit. So, if you find short-circuit current across \(ab\) at \(\Theta\) instead of open-circuit voltage across \(ab\), you can find Norton's Equivalent Circuit:

Let's set the voltage at node-\(y\) as \(V_{y}\). Apply KCL to node-\(y\):

\[
\frac{V_{y}}{10} + \frac{V_{y}}{40} + 2 + \frac{V_{y}}{4} = 0
\]

\[
V_{y} = -8.3 \, \text{V}
\]

\[
I_{n} = \frac{V_{y} = -8.3 \, \text{V}}{4 \Omega} = -2.075 \, \text{A}
\]
Example 3.

\[ \begin{align*}
\text{Find open-circuit voltage across } ab. \\
\text{Apply KVL. } 6i - 2i + 6i - 20 &= 0 \\
\Rightarrow i &= 2 \text{ A} \\
\text{Therefore } V_{ab} = 6i = 12 \text{ V}
\end{align*} \]

\[ \text{Find short-circuit current across } ab. \]

Using two mesh currents, we have

\[ \begin{align*}
-20 + 6i_1 - 2i + 6(i_1 - i_2) &= 0 \\
6(i_2 - i_1) + 10i_2 &= 0 \\
\Rightarrow i &= i_1 - i_2
\end{align*} \]

From these three equations, we obtain

\[ i_2 = \frac{120}{136} \text{ A} = I_{ab} \]

\[ \text{From } V_{ab} \text{ (open-circuit) and } I_{ab} \text{ (short-circuit), find } R_{\text{th}} \]

\[ R_{\text{th}} = \frac{V_{ab}}{I_{ab}} = \frac{12}{120/136} = 13.6 \text{ } \Omega \]
Therefore Thevenin's Equivalent Circuit is

\[ V_{in} = 12\, V \]
\[ R_{th} = 13.6\, \Omega \]

Norton's Equivalent Circuit is

\[ I_{n} = \frac{12}{136} \, A \]
\[ P_{n} = 13.6\, \Omega \]

\( \text{Example 4:} \)

Since the circuit has no independent source, \( i = 0 \) when we connect an open circuit to \( ab \).
Therefore \( V_{ab} = 0 \) and \( I_{ab} = 0 \) (open).

So, we cannot use \( R_{th} = \frac{V_{ab}}{I_{ab}} \) like Example 3.

So, we connect 1A test current source to \( ab \). Then we can say

\[ R_{th} = \frac{V_{ab}}{1\, A} \]

Let's set the minus node of the voltage source as ground for reference.
Apply KCL to node \( a \).

\[ \left( \frac{V_{ab} - 2i}{5} + \frac{V_{ab}}{10} \right) - 1 = 0 \]
and \( i = \frac{V_{ab}}{10} \)

Therefore \( V_{ab} - 2\left( \frac{V_{ab}}{10} \right) + \frac{V_{ab}}{10} - 1 = 0 \Rightarrow V_{ab} = \frac{50}{13} \, V \)

\[ : R_{th} = \frac{V_{ab}}{1} = \frac{10}{13} \, \Omega \]
Thevenin's Equivalent Circuit is

\[ R_{\text{th}} = 50 \Omega \]

Norton's Equivalent Circuit is the same as Thevenin's Circuit.