# Lecture 12

## P-N Junction Diodes: Part 2

## How do they work? (A little bit of math)

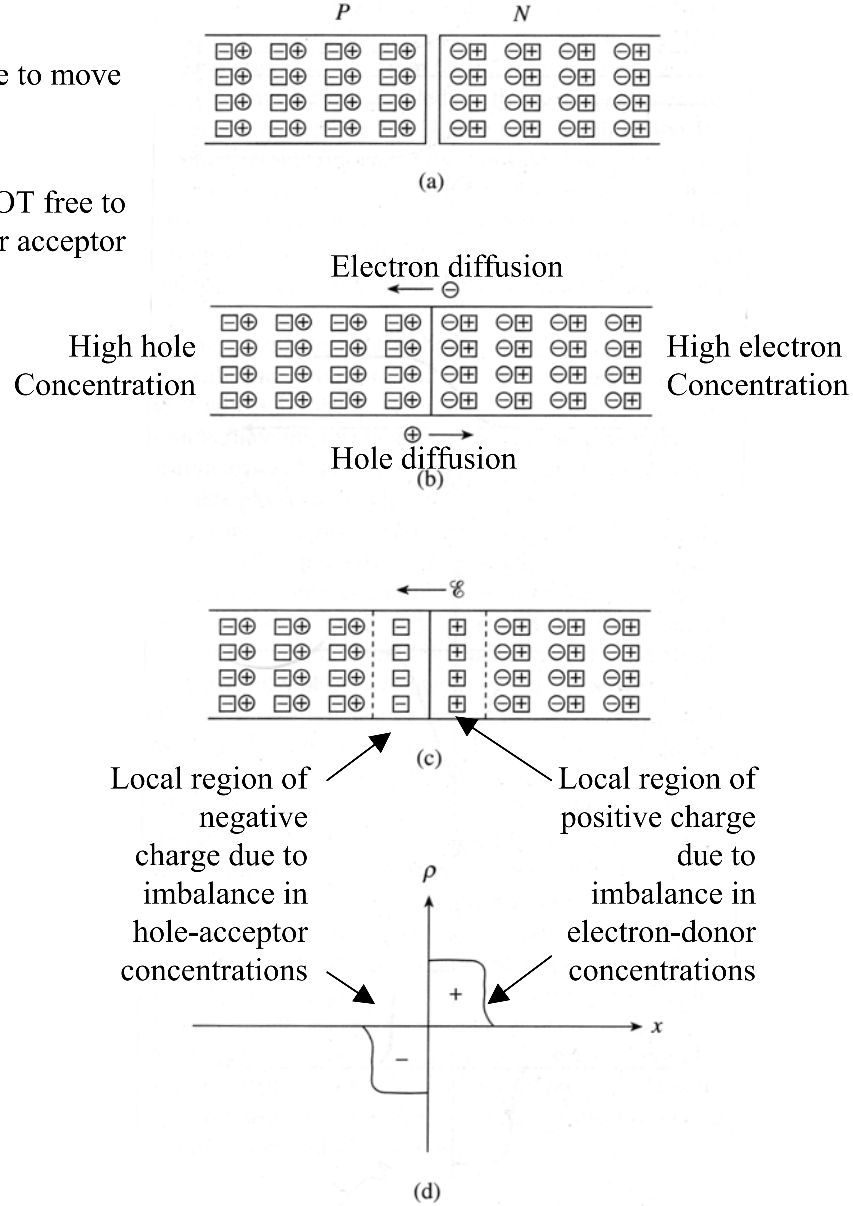
**Reading:**

**Pierret 6.1**

# Movement of electrons and holes when forming the junction

Circles are charges free to move (electrons and holes)

Squares are charges NOT free to move (ionized donor or acceptor atoms)




## Space Charge or Depletion Region

# Movement of electrons and holes when forming the junction

$$E = -dV/dx$$

$$-E\,dx = dV$$

$$-\int_{-x_p}^{x_n} E\,dx = \int_{V(-x_p)}^{V(x_n)} dV \quad = \quad V(x_n) - V(-x_p) = V_{bi}$$

*but...*

$$J_N = q\mu_n n E + qD_N \frac{dn}{dx} = 0$$

No net current flow in equilibrium

$$E = -\frac{D_N}{\mu_n}\frac{\frac{dn}{dx}}{n} = -\frac{kT}{q}\frac{\frac{dn}{dx}}{n}$$

*thus...*

$$V_{bi} = -\int_{-x_p}^{x_n} E\,dx = \frac{kT}{q}\int_{n(-x_p)}^{n(x_n)} \frac{\frac{dn}{dx}}{n} = \frac{kT}{q}\ln\left[\frac{n(x_n)}{n(-x_p)}\right]$$

# Movement of electrons and holes when forming the junction

$$V_{bi} = \frac{kT}{q}\ln\left[\frac{n(x_n)}{n(-x_p)}\right] = \frac{kT}{q}\ln\left[\frac{N_D}{n_i^2 / N_A}\right]$$

$$\boxed{V_{bi} = \frac{kT}{q}\ln\left[\frac{N_A N_D}{n_i^2}\right]}$$

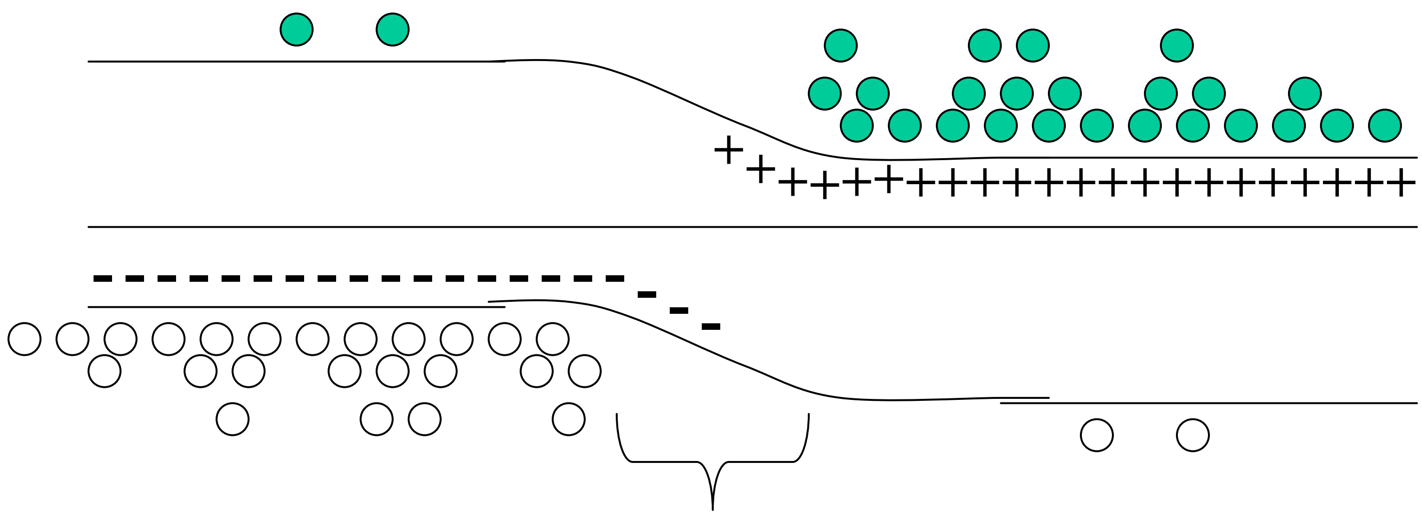
For $N_A=N_D=10^{15}/cm^{-3}$ in silicon at room temperature,
$V_{bi}$~0.6 V*

For a non-degenerate semiconductor, $|-qV_{bi}|<|E_g|$

*Note to those familiar with a diode turn on voltage: This is not the diode turn on voltage! This is the voltage required to reach a flat band diagram and sets an upper limit (typically an overestimate) for the voltage that can be applied to a diode before it burns itself up.

# Movement of electrons and holes when forming the junction

## Depletion Region Approximation



Depletion Region Approximation states that approximately no free carriers exist in the space charge region and no net charge exists outside of the depletion region (known as the quasi-neutral region). Thus,

$$\frac{dE}{dx} = \frac{\rho}{K_S \varepsilon_o} = \frac{q}{K_S \varepsilon_o}(p - n + N_D - N_A) = 0 \quad \textit{within the quasi-neutral region}$$

*becomes...*

$$\frac{dE}{dx} = \frac{q}{K_S \varepsilon_o}(N_D - N_A) \quad \textit{within the space charge region}$$

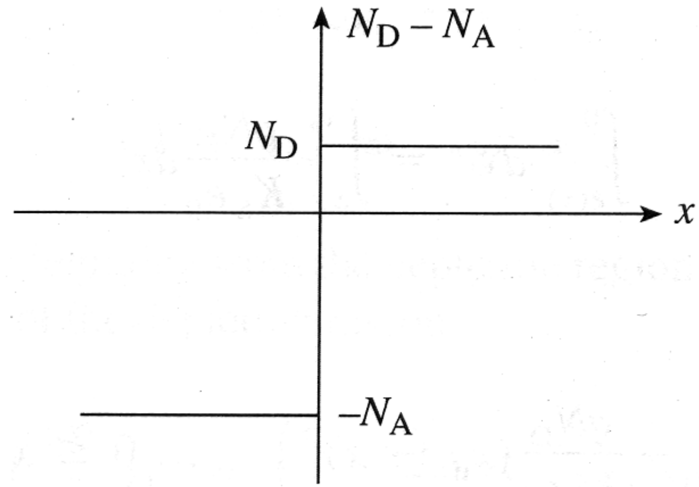
# Movement of electrons and holes when forming the junction

## Depletion Region Approximation: Step Junction Solution

$$\rho = \begin{cases} -qN_A & for\ -x_p \le x \le 0 \\ qN_D & for\ 0 \le x \le x_n \\ 0 & for\ x \le -x_p\ and\ x \ge x_n \end{cases}$$

*thus,*

$$\frac{dE}{dx} = \begin{cases} \dfrac{-qN_A}{K_S\varepsilon_o} & for\ -x_p \le x \le 0 \\ \dfrac{qN_D}{K_S\varepsilon_o} & for\ 0 \le x \le x_n \\ 0 & for\ x \le -x_p\ and\ x \ge x_n \end{cases}$$



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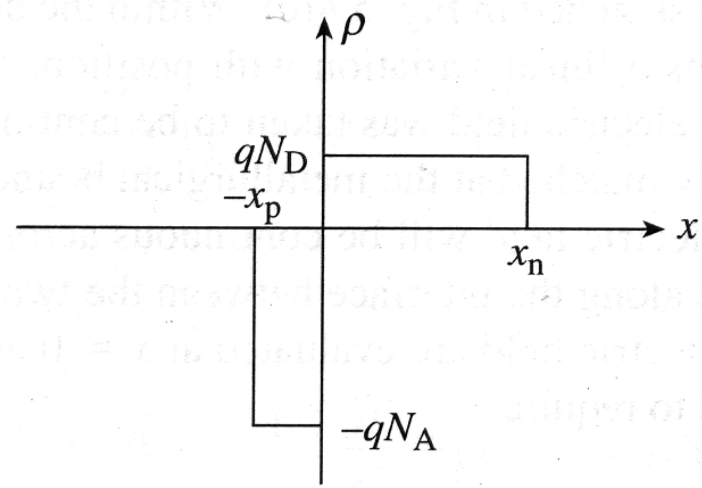



Where we have used:

$$\frac{dE}{dx} = \frac{\rho}{K_S\varepsilon_o}$$

# Movement of electrons and holes when forming the junction

## Depletion Region Approximation: Step Junction Solution

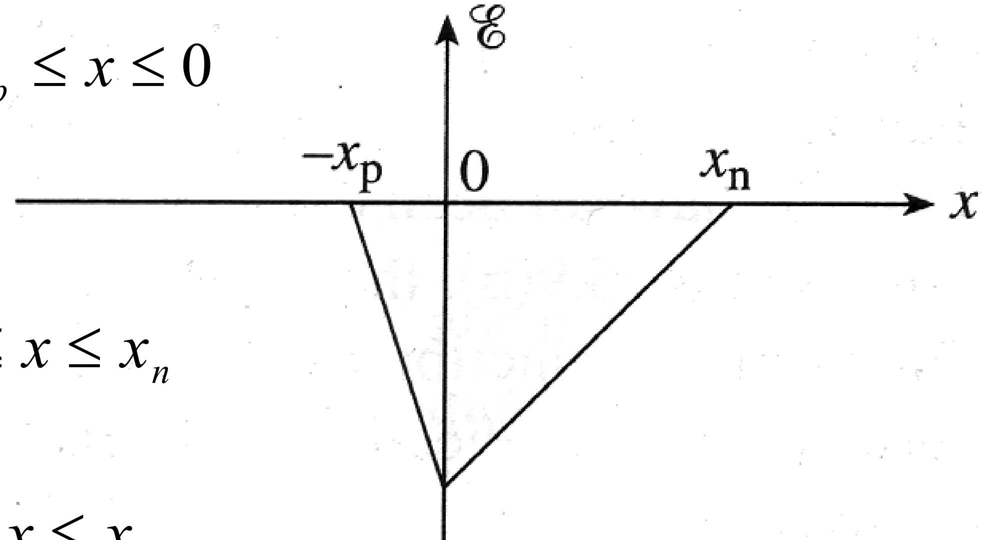
$$\int_0^{E(x)} dE' = \int_{-x_p}^{x} \frac{-qN_A}{K_S\varepsilon_o}dx' \quad for -x_p \le x \le 0$$

$$E(x) = \frac{-qN_A}{K_S\varepsilon_o}\left(x + x_p\right) \quad for -x_p \le x \le 0$$

*and*

$$\int_{E(x)}^{0} dE' = \int_{x}^{x_n} \frac{qN_D}{K_S\varepsilon_o}dx' \quad for\ 0 \le x \le x_n$$

$$E(x) = \frac{-qN_D}{K_S\varepsilon_o}\left(x_n - x\right) \quad for\ 0 \le x \le x_n$$



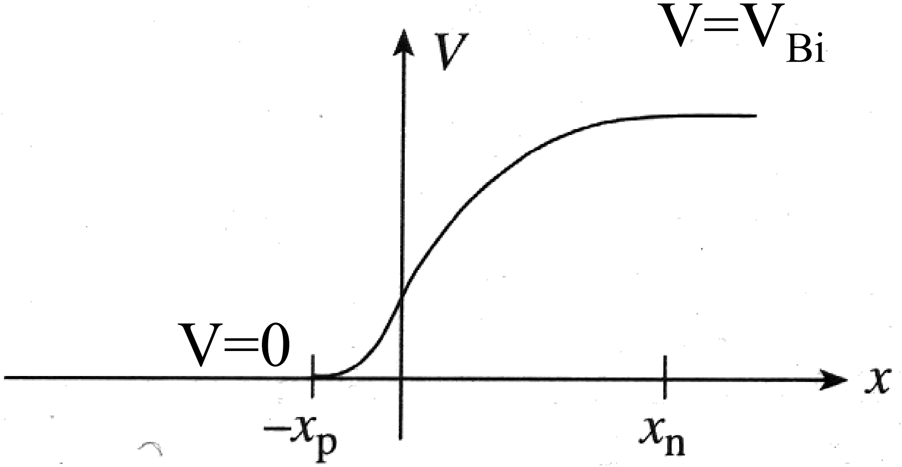

Since $E(x=0^-)=E(x=0^+)$

$$\boxed{N_A x_p = N_D x_n}$$

# Movement of electrons and holes when forming the junction

## Depletion Region Approximation: Step Junction Solution

$$E = -\frac{dV}{dx}$$




$$\frac{dV}{dx} = \begin{cases} \frac{qN_A}{K_S\varepsilon_o}\left(x_p + x\right) & for\ -x_p \le x \le 0 \\ \frac{qN_D}{K_S\varepsilon_o}\left(x_n - x\right) & for\ 0 \le x \le x_n \end{cases}$$

*or,*

$$\int_0^{V(x)} dV' = \int_{-x_p}^{x} \frac{qN_A}{K_S\varepsilon_o}\left(x_p + x'\right)dx' \quad for\ -x_p \le x \le 0$$

$$\int_{V(x)}^{V_{Bi}} dV' = \int_{x}^{x_n} \frac{qN_D}{K_S\varepsilon_o}\left(x_n - x'\right)dx' \quad for\ 0 \le x \le x_n$$

$$V(x) = \begin{cases} \frac{qN_A}{2K_S\varepsilon_o}\left(x_p + x\right)^2 & for\ -x_p \le x \le 0 \\ V_{bi} - \frac{qN_D}{2K_S\varepsilon_o}\left(x_n - x\right)^2 & for\ 0 \le x \le x_n \end{cases}$$

# Movement of electrons and holes when forming the junction

## Depletion Region Approximation: Step Junction Solution

At x=0,

$$\frac{qN_A}{2K_S\varepsilon_o}\left(x_p\right)^2 = V_{bi} - \frac{qN_D}{2K_S\varepsilon_o}\left(x_n\right)^2$$

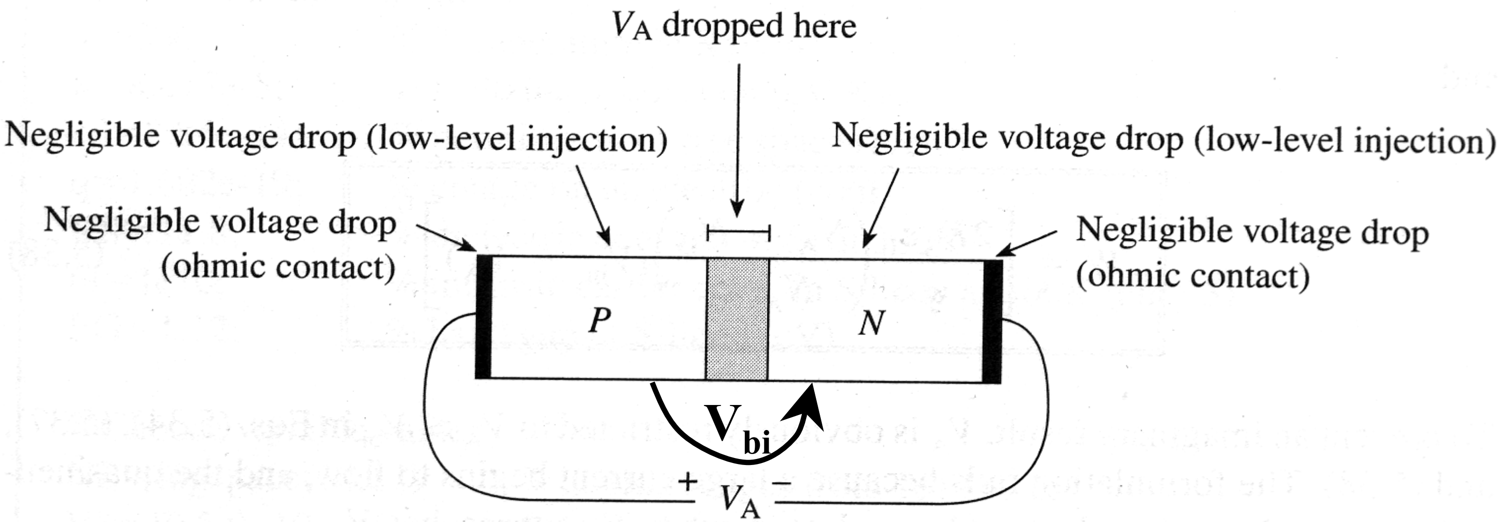
$$U\sin g,\ x_p = \left(x_n N_D\right)\Big/ N_A$$

$$x_n = \sqrt{\frac{2K_S\varepsilon_o}{q}\frac{N_A}{N_D\left(N_A+N_D\right)}V_{bi}} \quad and \quad x_p = \sqrt{\frac{2K_S\varepsilon_o}{q}\frac{N_D}{N_A\left(N_A+N_D\right)}V_{bi}}$$

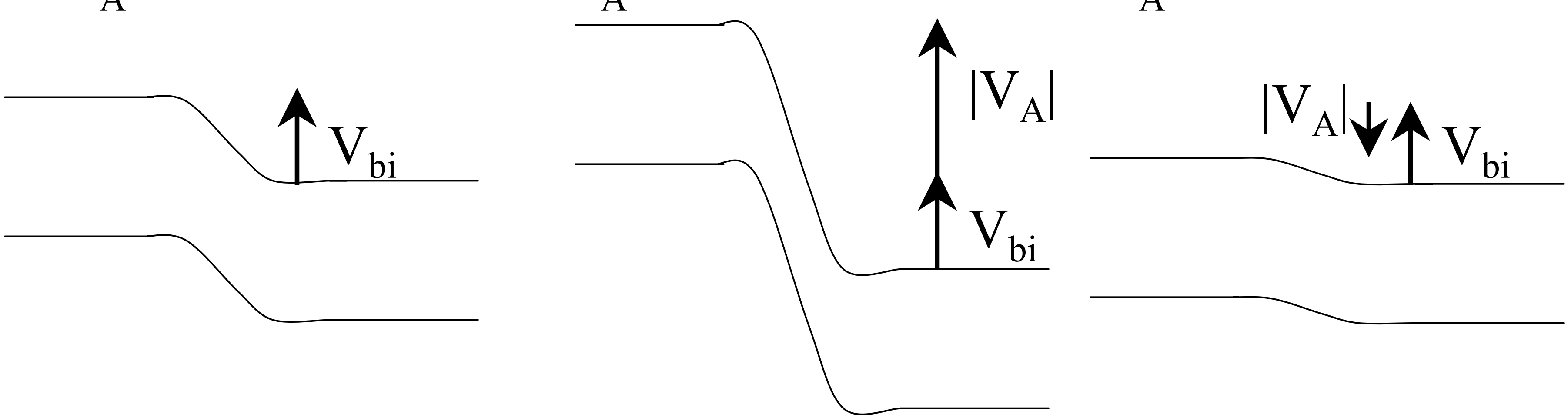
$$W = x_p + x_n = \sqrt{\frac{2K_S\varepsilon_o}{q}\frac{\left(N_A+N_D\right)}{N_A N_D}V_{bi}}$$

# Movement of electrons and holes when forming the junction

## Depletion Region Approximation: Step Junction Solution




$V_A=0$ : No Bias    $V_A<0$ : Reverse Bias    $V_A>0$ : Forward Bias



# Movement of electrons and holes when forming the junction

## Depletion Region Approximation: Step Junction Solution

Thus, only the boundary conditions change resulting in direct replacement of $V_{bi}$ with $(V_{bi}-V_A)$

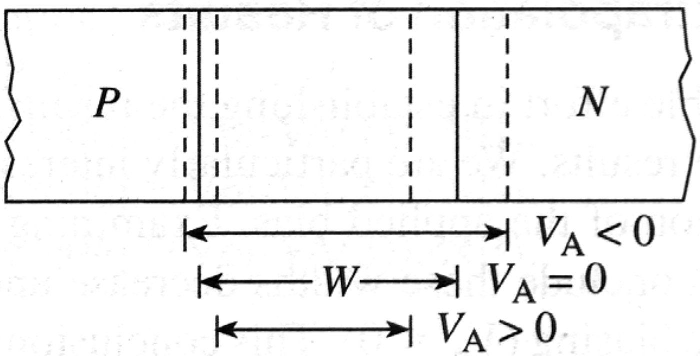
$$x_n = \sqrt{\frac{2K_S\varepsilon_o}{q}\frac{N_A}{N_D\left(N_A+N_D\right)}\left(V_{bi}-V_A\right)} \quad and \quad x_p = \sqrt{\frac{2K_S\varepsilon_o}{q}\frac{N_D}{N_A\left(N_A+N_D\right)}\left(V_{bi}-V_A\right)}$$

$$\boxed{W = x_p + x_n = \sqrt{\frac{2K_S\varepsilon_o}{q}\frac{\left(N_A+N_D\right)}{N_A N_D}\left(V_{bi}-V_A\right)}}$$

# Movement of electrons and holes when forming the junction

## Step Junction Solution: What does it mean?

Consider a $p^+$ -n junction (heavily doped p-side, normal or lightly doped n side).

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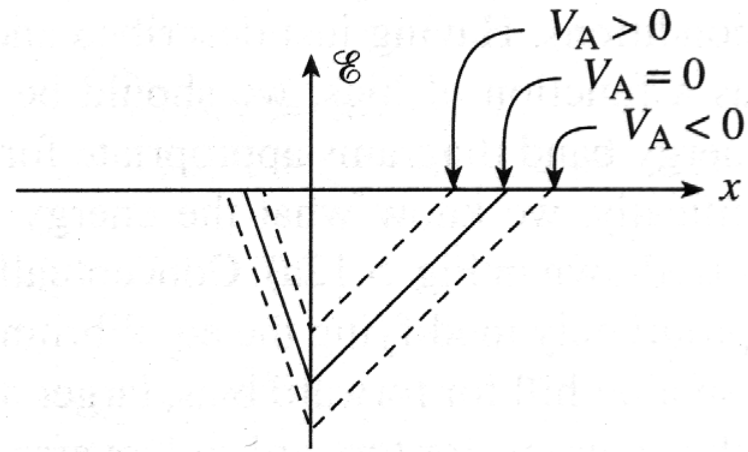


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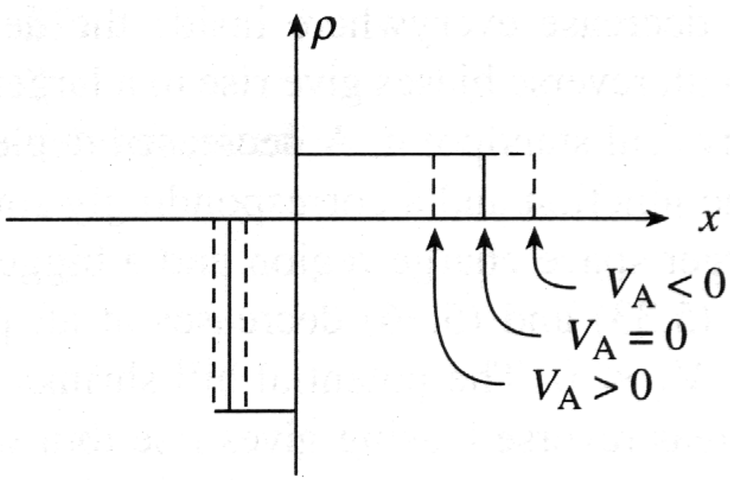
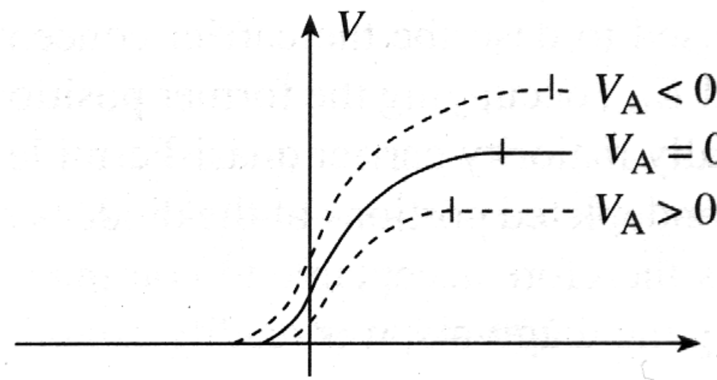
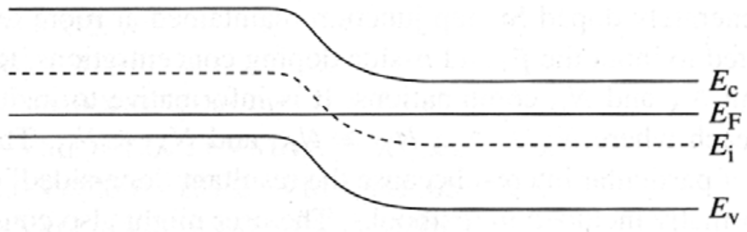


# Movement of electrons and holes when forming the junction

Step Junction Solution: What does it mean?

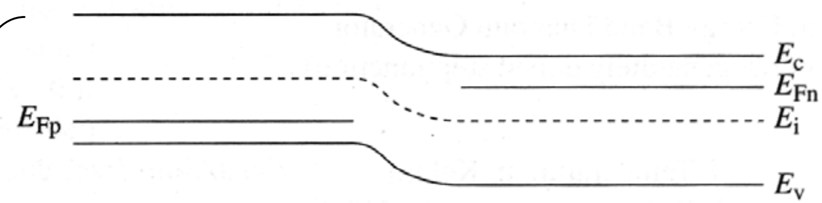
**Fermi-level only applies to equilibrium (no current flowing)**



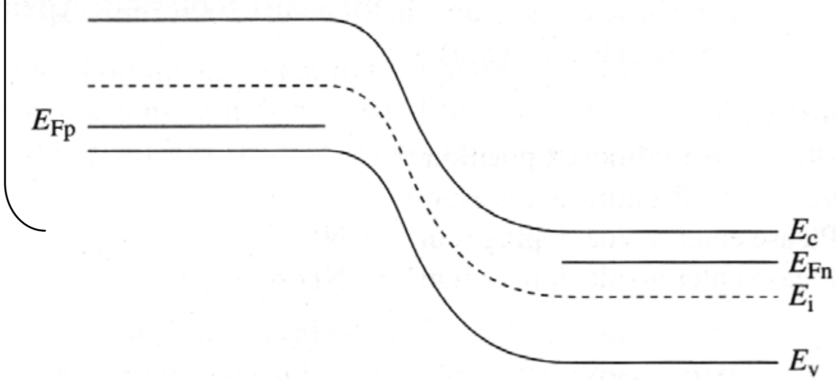

(a) Equilibrium ($V_A = 0$)

**Majority carrier Quasi-fermi levels**

$$\underbrace{E_{fp}-E_{fn}}=-qV_A$$




(b) Forward bias ($V_A > 0$)




(c) Reverse bias ($V_A < 0$)