Lecture 14

P-N Junction Diodes: Part 3

Quantitative Analysis (Math, math and more math)

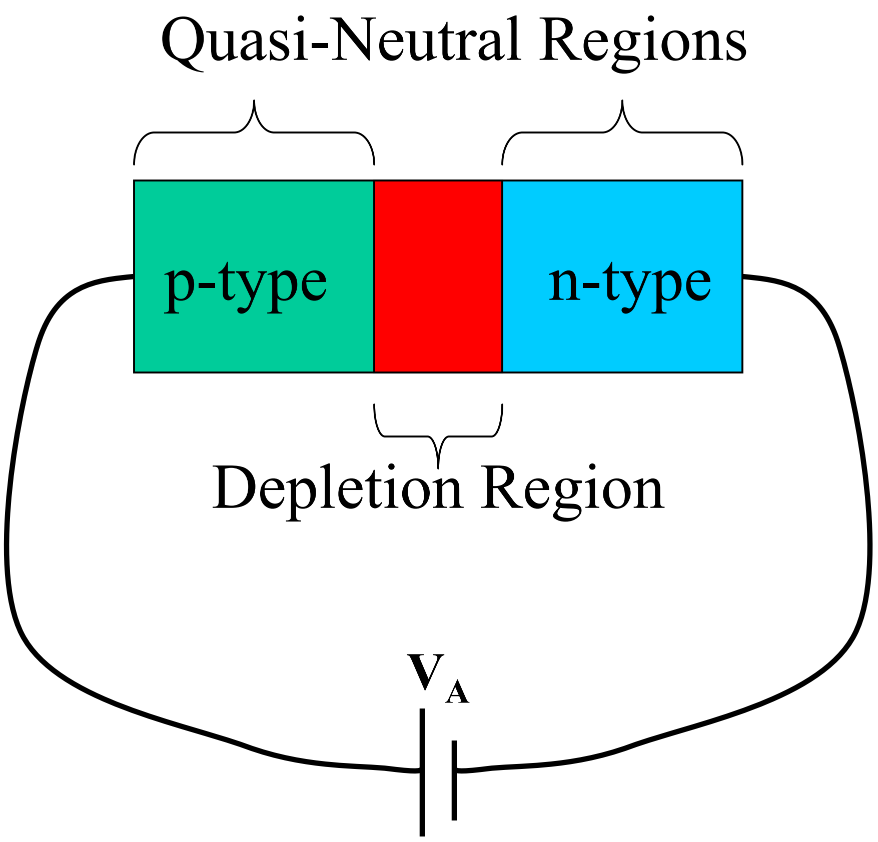
Reading:

Pierret 6.1

Quantitative p-n Diode Solution

Assumptions:

1) steady state conditions
2) non- degenerate doping
3) one- dimensional analysis
4) low- level injection
5) no light ($G_L = 0$)



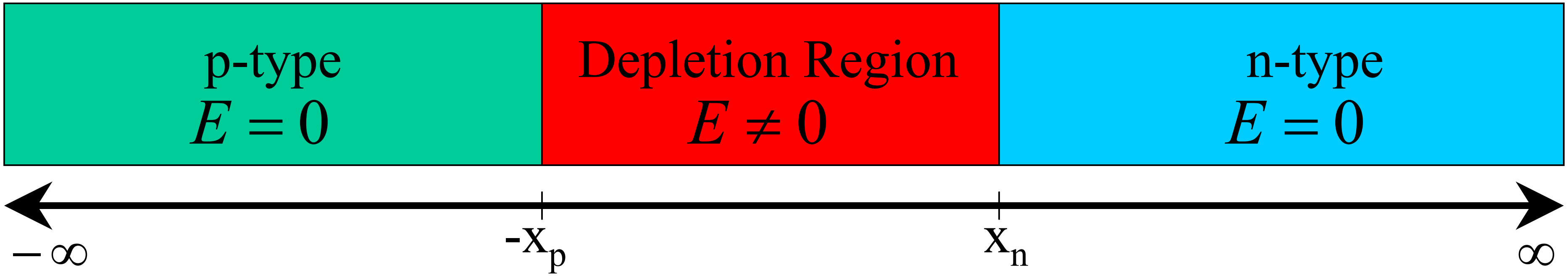
Current equations:

$J = J_p(x) + J_n(x)$

$J_n = q\,\mu_n n E + q D_n (dn/dx)$

$J_p = q\,\mu_p p E - q D_p\,(dp/dx)$

Quantitative p-n Diode Solution



Application of the Minority Carrier Diffusion Equation

$$\frac{\partial(\Delta n_p)}{\partial t} = D_N \frac{\partial^2(\Delta n_p)}{\partial x^2} - \frac{(\Delta n_p)}{\tau_n} + G_L$$

$$0 = D_N \frac{\partial^2(\Delta n_p)}{\partial x^2} - \frac{(\Delta n_p)}{\tau_n}$$

Since electric fields exist in the depletion region, the minority carrier diffusion equation does not apply here.

$$\frac{\partial(\Delta p_n)}{\partial t} = D_P \frac{\partial^2(\Delta p_n)}{\partial x^2} - \frac{(\Delta p_n)}{\tau_p} + G_L$$

$$0 = D_P \frac{\partial^2(\Delta p_n)}{\partial x^2} - \frac{(\Delta p_n)}{\tau_p} + 0$$

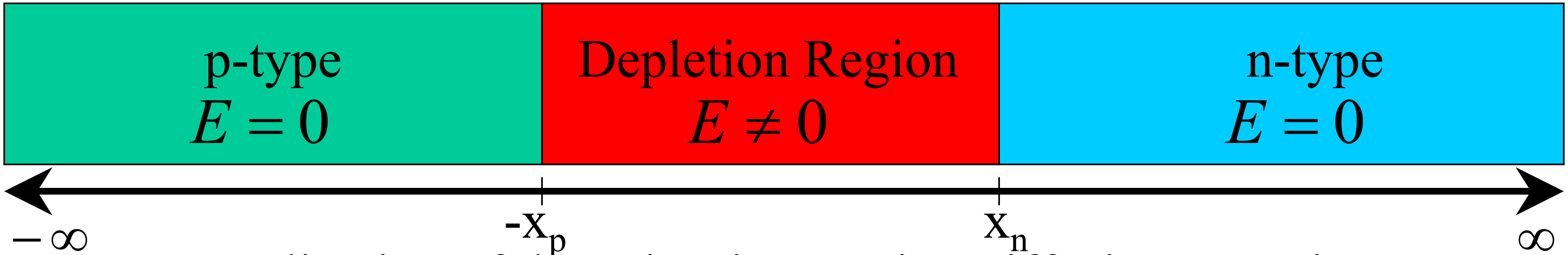
Boundary Condition :
$$\Delta n_p(x \rightarrow -\infty) = 0$$

Boundary Condition :
$$\Delta n_p(x = -x_p) = ?$$

Boundary Condition :
$$\Delta p_n(x = x_n) = ?$$

Boundary Condition :
$$\Delta p_n(x \rightarrow \infty) = 0$$

Quantitative p-n Diode Solution



Application of the Minority Carrier Diffusion Equation

Boundary Condition :

$$\Delta n_p(x = -x_p) = ?$$

Boundary Condition :

$$\Delta p_n(x = x_n) = ?$$

$$n = n_i e^{(F_N - E_i)/kT} \quad \text{and} \quad p = n_i e^{(E_i - F_P)/kT}$$

$$n_p(x = -x_p) p_p(x = -x_p) = n_i^2 e^{(F_N - F_P)/kT}$$

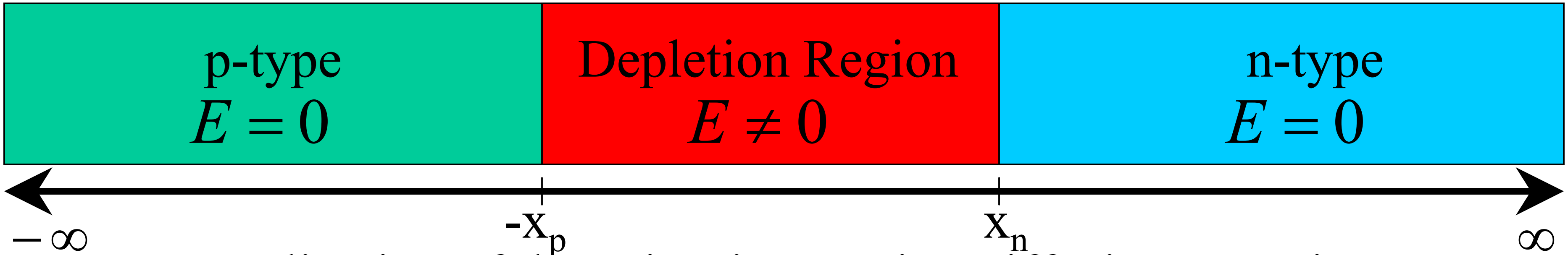
$$n_p(x = -x_p) p_p(x = -x_p) = n_p(x = -x_p) N_A = n_i^2 e^{qV_A/kT}$$

$$n_p(x = -x_p) = \frac{n_i^2}{N_A} e^{qV_A/kT}$$

$$\Delta n_p(x = -x_p) = \frac{n_i^2}{N_A} e^{qV_A/kT} - n_o$$

$$\Delta n_p(x = -x_p) = \frac{n_i^2}{N_A}\left(e^{qV_A/kT} - 1\right) \quad \text{and similarly at } x = x_n \quad \Delta p_n(x = x_n) = \frac{n_i^2}{N_D}\left(e^{qV_A/kT} - 1\right)$$

Quantitative p-n Diode Solution



Application of the Minority Carrier Diffusion Equation

Boundary Condition :

$$\Delta n_p(x=-x_p)=?$$

Boundary Condition :

$$\Delta p_n(x=x_n)=?$$

$$n = n_i e^{(F_N - E_i)/kT} \quad and \quad p = n_i e^{(E_i - F_P)/kT}$$

$$n_p(x=-x_p)\,p_p(x=-x_p) = n_i^2 e^{(F_N - F_P)/kT}$$

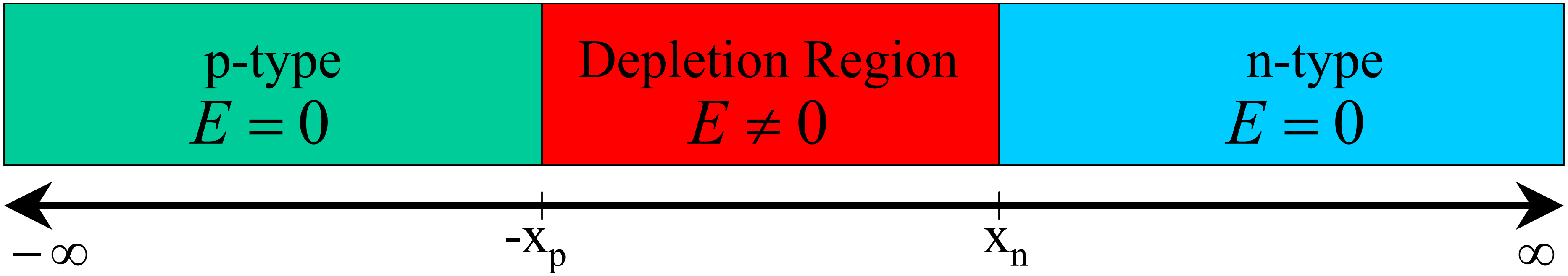
$$n_p(x=-x_p)\,p_p(x=-x_p) = n_p(x=-x_p)\,N_A = n_i^2 e^{qV_A/kT}$$

$$n_p(x=-x_p) = \frac{n_i^2}{N_A} e^{qV_A/kT}$$

$$\Delta n_p(x=-x_p) = \frac{n_i^2}{N_A} e^{qV_A/kT} - n_o$$

$$\Delta n_p(x=-x_p) = \frac{n_i^2}{N_A}\left(e^{qV_A/kT} - 1\right) \quad and\ similarly\ at\ x = x_n \quad \Delta p_n(x=x_n) = \frac{n_i^2}{N_D}\left(e^{qV_A/kT} - 1\right)$$

Quantitative p-n Diode Solution



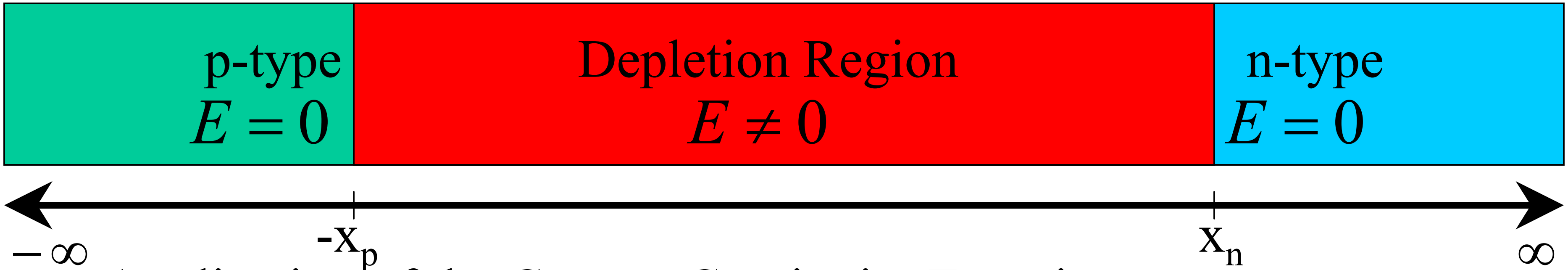
Application of the Current Continuity Equation

$$J_n = q\left(\mu_n nE + D_n \frac{dn}{dx}\right)$$
$$= qD_n \frac{d(n_o + \Delta n_p)}{dx}$$
$$= qD_n \frac{d\Delta n_p}{dx}$$

?

$$J_p = q\left(\mu_p pE - D_p \frac{dp}{dx}\right)$$
$$= -qD_p \frac{d(p_o + \Delta p_n)}{dx}$$
$$= -qD_p \frac{d\Delta p_n}{dx}$$

Quantitative p-n Diode Solution



Application of the Current Continuity Equation: Depletion Region

$$\frac{\partial n}{\partial t} = \frac{1}{q}\nabla \cdot J_N + \left.\frac{\partial n}{\partial t}\right|_{Recombination-Generation} + \left.\frac{\partial n}{\partial t}\right|_{\substack{All\ other\ processes \\ such\ as\ light,\ etc...}}$$

$$0 = \frac{1}{q}\nabla \cdot J_N$$

$$0 = \frac{1}{q}\frac{\partial J_N}{\partial x}$$

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\nabla \cdot J_P + \left.\frac{\partial p}{\partial t}\right|_{Recombination-Generation} + \left.\frac{\partial p}{\partial t}\right|_{\substack{All\ other\ processes \\ such\ as\ light,\ etc...}}$$

$$0 = -\frac{1}{q}\nabla \cdot J_P$$

$$0 = -\frac{1}{q}\frac{\partial J_P}{\partial x}$$

No thermal recombination and generation implies J_n and J_p are constant throughout the depletion region. Thus, the total current can be define in terms of only the current at the depletion region edges.

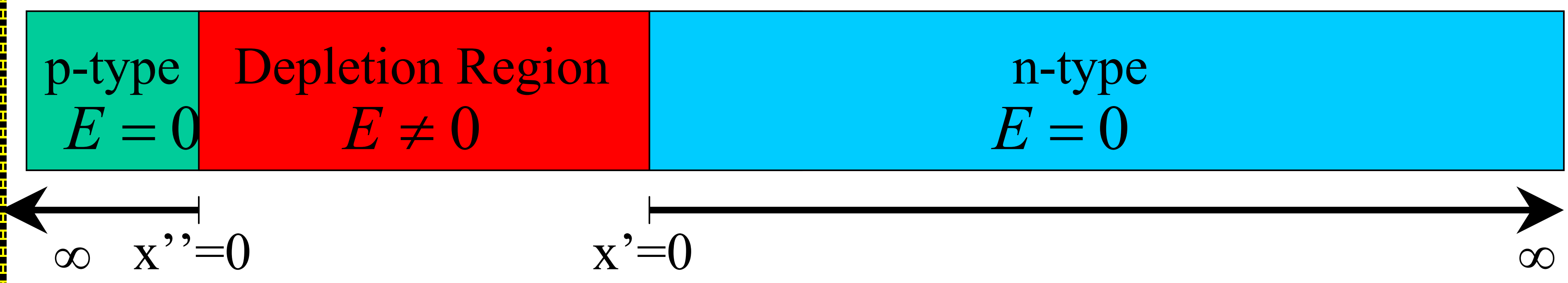
$$J = J_n(-x_p) + J_p(x_n)$$

Quantitative p-n Diode Solution

Approach:

- Solve minority carrier diffusion equation in quasi-neutral regions
- Determine minority carrier currents from continuity equation
- Evaluate currents at the depletion region edges
- Add these together and multiply by area to determine the total current through the device.
- Use translated axes, $x \rightarrow x'$ and $-x \rightarrow x''$ in our solution.

Quantitative p-n Diode Solution



$$0 = D_P \frac{\partial^2 (\Delta p_n)}{\partial x'^2} - \frac{(\Delta p_n)}{\tau_p}$$

$$\Delta p_n(x') = A e^{(-x'/L_P)} + B e^{(+x'/L_P)} \quad where \quad L_P \equiv \sqrt{D_p \tau_p}$$

Boundary Conditions:

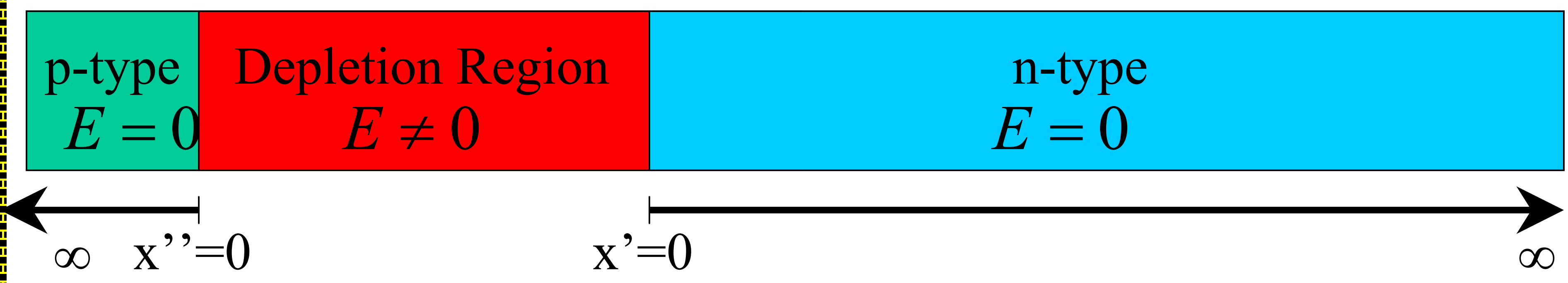
$$\Delta p_n(x' \rightarrow \infty) = 0$$

$$\Delta p_n(x'=0) = \frac{n_i^2}{N_D}\left(e^{qV_A/kT} - 1\right)$$

$$B = 0 \quad and \quad A = \Delta p_n(x'=0) = \frac{n_i^2}{N_D}\left(e^{qV_A/kT} - 1\right)$$

$$\boxed{\Delta p_n(x') = \frac{n_i^2}{N_D}\left(e^{qV_A/kT} - 1\right) e^{(-x'/L_P)} \quad for\ x' \geq 0}$$

Quantitative p-n Diode Solution

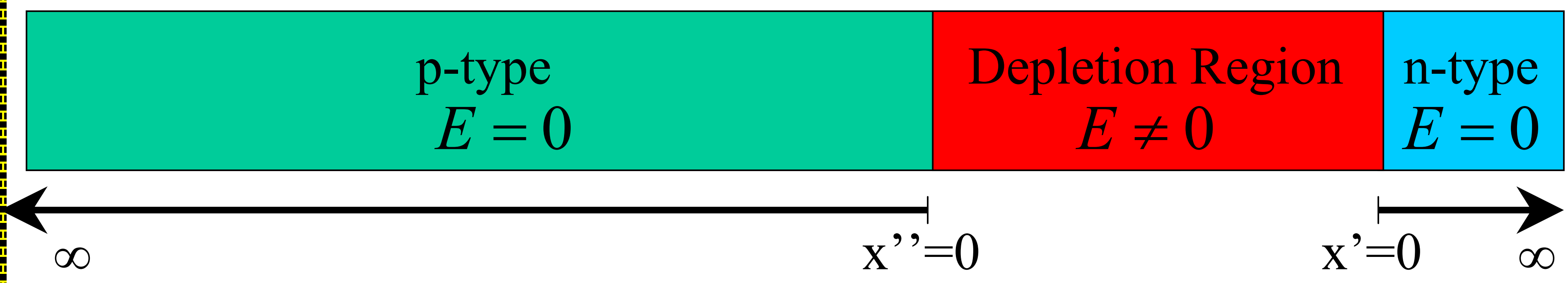


$$\Delta p_n(x') = \frac{n_i^2}{N_D}\left(e^{qV_A/kT} - 1\right)e^{(-x'/L_P)} \quad for\ x' \geq 0$$

$$J_p = -qD_p \frac{d\Delta p_n}{dx}$$

$$J_p = q\frac{D_p n_i^2}{L_p N_D}\left(e^{qV_A/kT} - 1\right)e^{(-x'/L_P)} \quad for\ x' \geq 0$$

Quantitative p-n Diode Solution



Similarly for electrons on the p-side…

$$\Delta n_p(x'') = \frac{n_i^2}{N_A}\left(e^{qV_A/kT} - 1\right)e^{(-x''/L_n)} \quad for\ x'' \geq 0$$

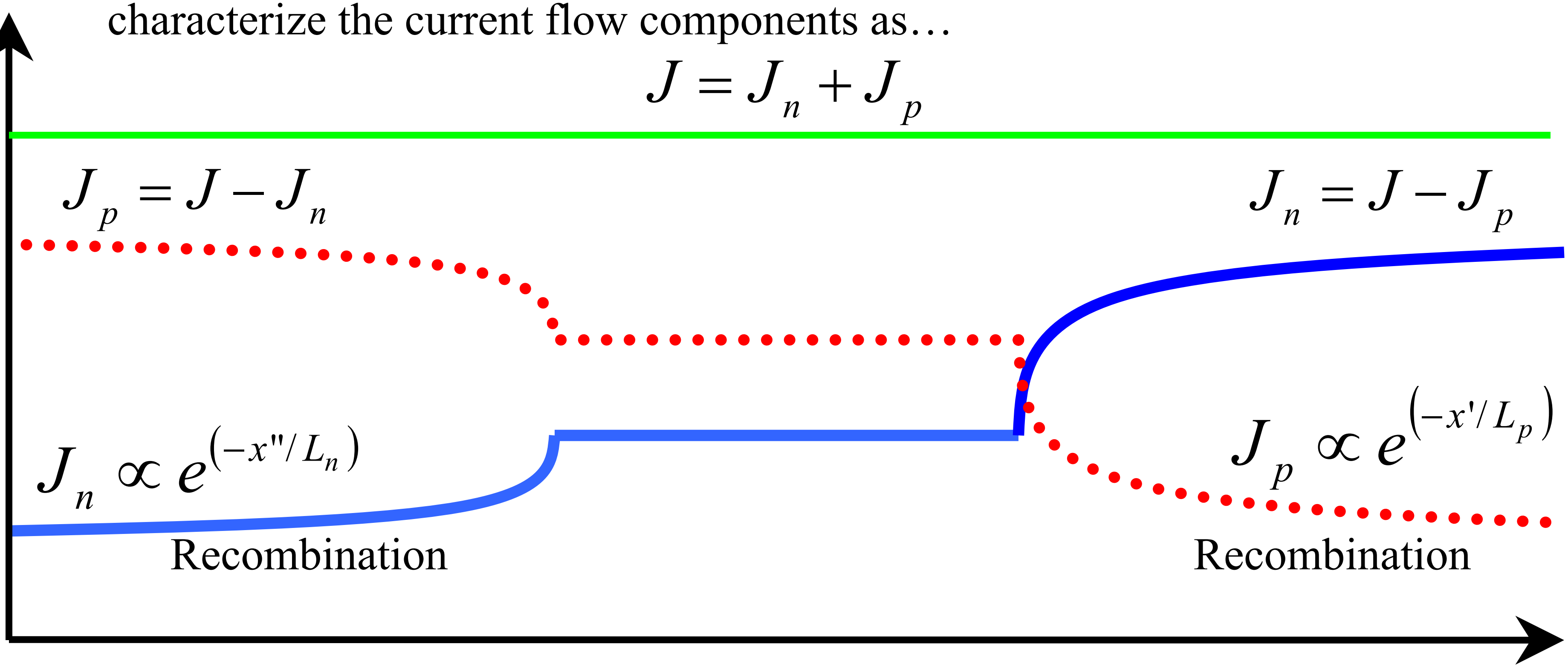
$$J_n = -qD_n \frac{d\Delta n_p}{dx}$$

$$J_n = q\frac{D_n n_i^2}{L_n N_A}\left(e^{qV_A/kT} - 1\right)e^{(-x''/L_n)} \quad for\ x'' \geq 0$$

Quantitative p-n Diode Solution

Total on current is constant throughout the device. Thus, we can characterize the current flow components as…

$$J = J_n + J_p$$



Quantitative p-n Diode Solution

Thus, evaluating the current components at the depletion region edges, we have…

$$J = J_n(x''=0) + J_p(x'=0) = J_n(x''=0) + J_p(x''=0) = J_n(x'=0) + J_p(x'=0)$$

$$J = q\left(\frac{D_n n_i^2}{L_n N_A} + \frac{D_p n_i^2}{L_p N_D}\right)\left(e^{qV_A/kT} - 1\right) \quad \textit{for all } x$$

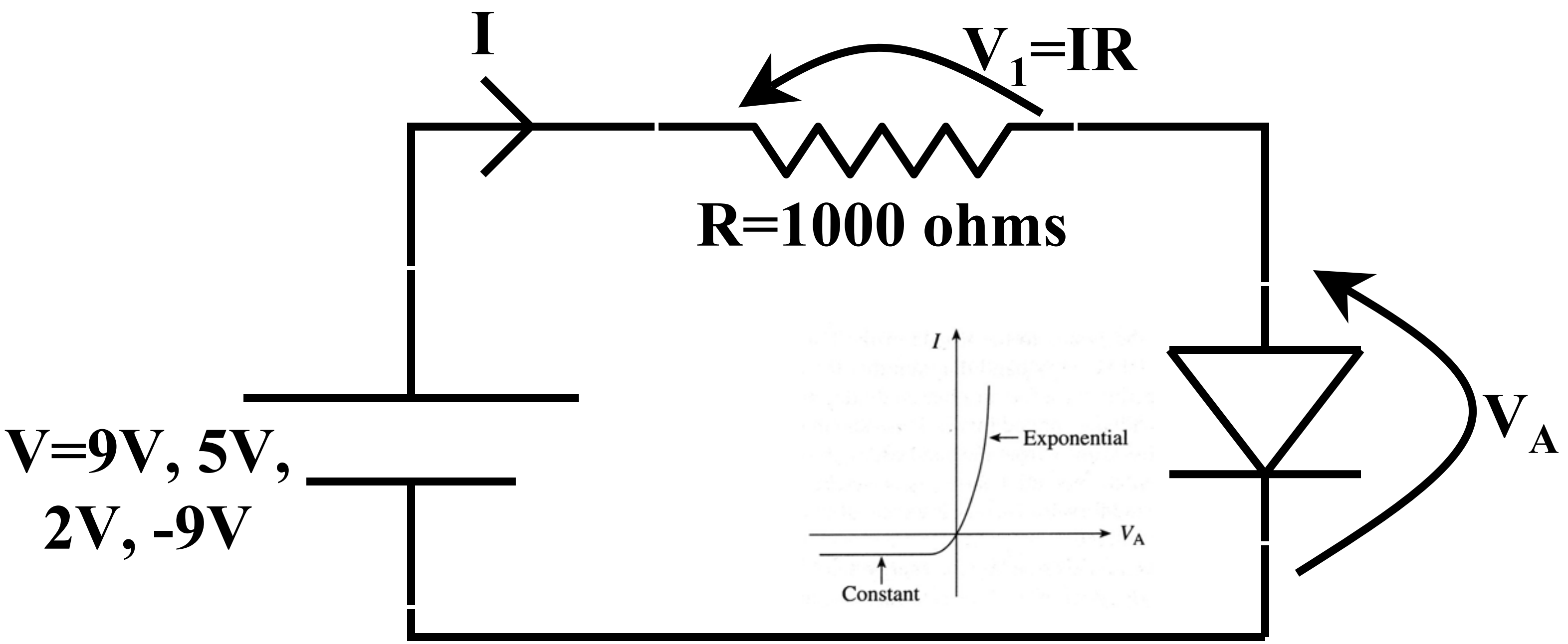
or

$$I = I_o\left(e^{qV_A/kT} - 1\right) \quad \textit{where } I_o = qA\left(\frac{D_n n_i^2}{L_n N_A} + \frac{D_p n_i^2}{L_p N_D}\right)$$

I_o *is the "reverse saturation current"*

Note: Vref from our previous qualitative analysis equation is the thermal voltage, kT/q

Quantitative p-n Diode Solution
Examples: Diode in a circuit



$$9V = I(1000) + V_A$$

$$I = 1e-12\left(e^{V_A/0.0259V} - 1\right)$$

or

$$9V = \left[1e-12\left(e^{V_A/0.0259V} - 1\right)\right](1000) + V_A$$

$$9V = 1e-9\left(e^{V_A/0.0259V} - 1\right) + V_A$$

$$I = I_o\left(e^{qV_A/kT} - 1\right) \quad where\ I_o = 1\ pA$$

Solutions

V	V_A	I
9V	0.59V	8.4 mA
5V	0.58V	4.4 mA
2V	0.55V	1.5 mA
-9V	-9.0V	-1 pA

In forward bias (V_A>0) the V_A is ~constant for large differences in current

In reverse bias (V_A<0) the current is ~constant (=saturation current)