Mostly Optional

Lecture 18

Bipolar Junction Transistors (BJT): Part 2

Quantitative Understanding - Do the math!

Reading:

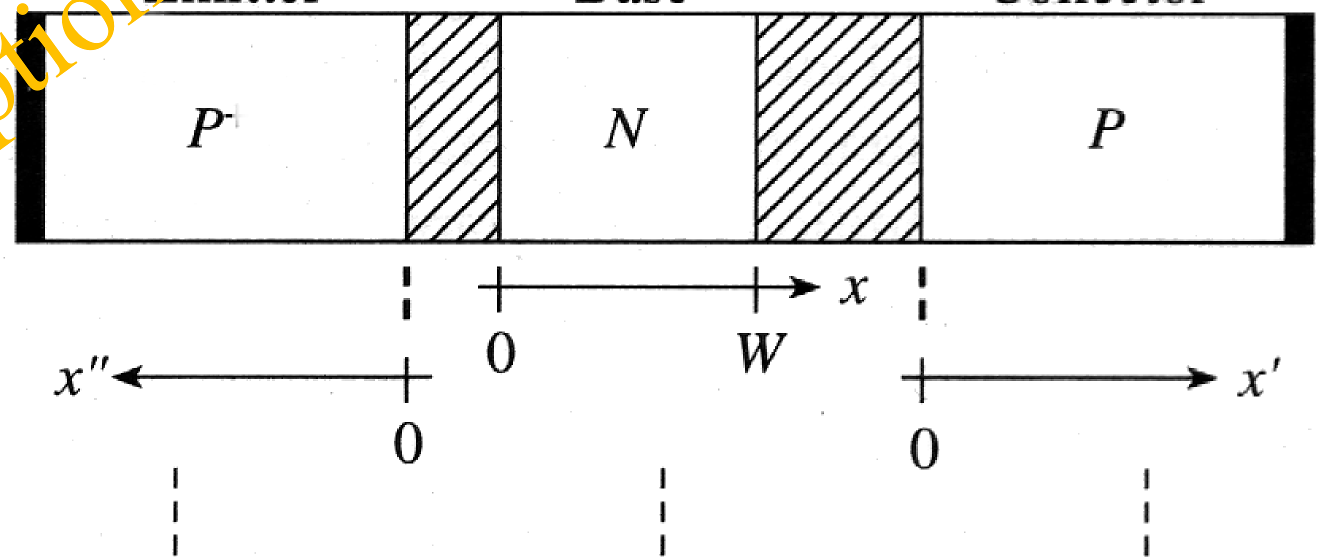
Pierret 11.1

Due to the shortened class period, we will not formally derive the Ebers Moll (Large Signal) model of the BJT. See the next lecture for a phenomenological development of the Ebers Moll Model.

Bipolar Junction Transistor (BJT) Quantitative Solution

The Definitions...

Optional



$N_E = N_{AE}$ $\quad$ $N_B = N_{DB}$ $\quad$ $N_C = N_{AC}$

$D_E = D_N$ $\quad$ $D_B = D_P$ $\quad$ $D_C = D_N$

$\tau_E = \tau_n$ $\quad$ $\tau_B = \tau_p$ $\quad$ $\tau_C = \tau_n$

$L_E = L_N$ $\quad$ $L_B = L_P$ $\quad$ $L_C = L_N$

$n_{E0} = n_{p0} = n_i^2/N_E$ $\quad$ $p_{B0} = p_{n0} = n_i^2/N_B$ $\quad$ $n_{C0} = n_{p0} = n_i^2/N_C$

Bipolar Junction Transistor (BJT) Quantitative Solution

The Problem Setup...

Optional

Emitter Region

The diffusion equation to be solved is

$$0 = D_E \frac{d^2 \Delta n_E}{dx''^2} - \frac{\Delta n_E}{\tau_E}$$

subject to the boundary conditions

$$\Delta n_E(x'' \to \infty) = 0$$

$$\boxed{\Delta n_E(x'' = 0) = n_{E0}(e^{qV_{EB}/kT} - 1)}$$

Base Region

The diffusion equation to be solved is

$$0 = D_B \frac{d^2 \Delta p_B}{dx^2} - \frac{\Delta p_B}{\tau_B}$$

subject to the boundary conditions

Will use these definitions on slides 7 & 11

$$\boxed{\begin{aligned}\Delta p_B(0) &= p_{B0}(e^{qV_{EB}/kT} - 1)\\ \Delta p_B(W) &= p_{B0}(e^{qV_{CB}/kT} - 1)\end{aligned}}$$

Collector Region

The diffusion equation to be solved is

$$0 = D_C \frac{d^2 \Delta n_C}{dx'^2} - \frac{\Delta n_C}{\tau_C}$$

subject to the boundary conditions

$$\Delta n_C(x' \to \infty) = 0$$

$$\boxed{\Delta n_C(x' = 0) = n_{C0}(e^{qV_{CB}/kT} - 1)}$$

Then Use

$$I_{En} = -qAD_E \left.\frac{d\Delta n_E}{dx''}\right|_{x''=0}$$

$$I_{Ep} = -qAD_B \left.\frac{d\Delta p_B}{dx}\right|_{x=0}$$

$$I_{Cp} = -qAD_B \left.\frac{d\Delta p_B}{dx}\right|_{x=W}$$

$$I_{Cn} = qAD_C \left.\frac{d\Delta n_C}{dx'}\right|_{x'=0}$$

LAW OF THE JUNCTION!

From our diode boundary conditions

Bipolar Junction Transistor (BJT) Quantitative Solution

The Problem Setup...

Optional

$$I_{En} = -qAD_E \left. \frac{d\Delta n_E}{dx''} \right|_{x''=0}$$

$$I_{Ep} = -qAD_B \left. \frac{d\Delta p_B}{dx} \right|_{x=0}$$

$$I_{Cp} = -qAD_B \left. \frac{d\Delta p_B}{dx} \right|_{x=W}$$

$$I_{Cn} = qAD_C \left. \frac{d\Delta n_C}{dx'} \right|_{x'=0}$$

Then Use

$$\gamma = \frac{I_{Ep}}{I_{Ep} + I_{En}}$$

$$\alpha_T = \frac{I_{Cp}}{I_{Ep}}$$

$$\alpha_{dc} = \gamma \alpha_T$$

$$\beta_{dc} = \frac{\alpha_{dc}}{1 - \alpha_{dc}}$$

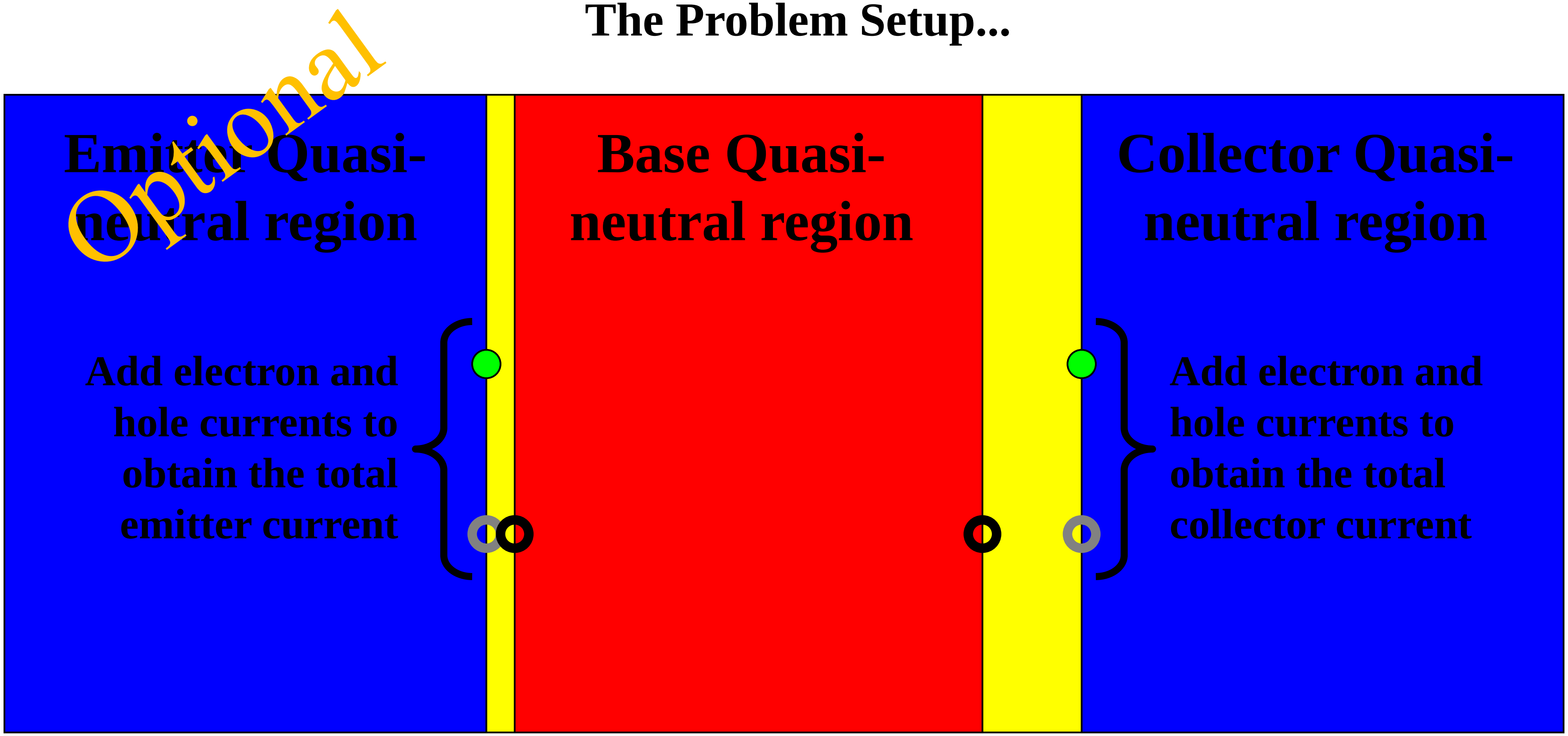
$$I_E = I_{Ep} + I_{En}$$

$$I_C = I_{Cp} + I_{Cn}$$

$$I_B = I_E - I_C$$

Bipolar Junction Transistor (BJT) Quantitative Solution

The Problem Setup...



Find Hole Current here…

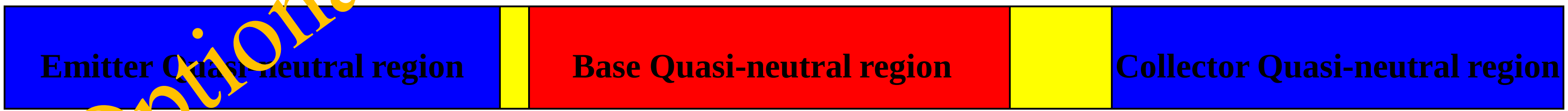
… will equal hole current here due to no recombination-generation in depletion regions

… add this current to the electron current here to determine the total current in the emitter and collector

Bipolar Junction Transistor (BJT) Quantitative Solution

The problem solution in the emitter and collector quasi-neutral regions...

Optional



$$\Delta n_E(x'') = A_1 e^{-x''/L_E} + A_2 e^{x''/L_E}$$

$$\Delta n_E(x'') = n_{E0}\left(e^{qV_{EB}/kT} - 1\right)e^{-x''/L_E}$$

$$\Delta n_C(x') = n_{C0}\left(e^{qV_{CB}/kT} - 1\right)e^{-x'/L_C}$$

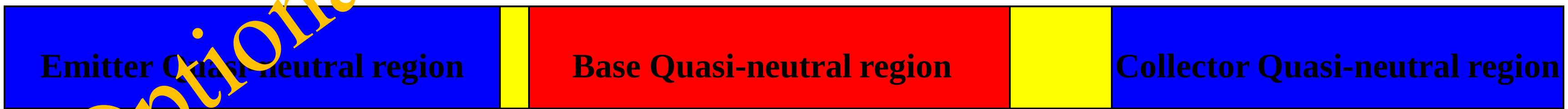
$$\boxed{I_{En} = qA\frac{D_E}{L_E}n_{E0}\left(e^{qV_{EB}/kT} - 1\right)}$$

$$\boxed{I_{Cn} = -qA\frac{D_C}{L_C}n_{C0}\left(e^{qV_{CB}/kT} - 1\right)}$$

Bipolar Junction Transistor (BJT) Quantitative Solution

The problem solution in the base quasi-neutral region...

Optional



$$\Delta p_B(x) = A_1 e^{-x/L_B} + A_2 e^{x/L_B}$$

$$\Delta p_B(0) = p_{B0}(e^{qV_{EB}/kT} - 1) = A_1 + A_2$$

$$\Delta p_B(W) = p_{B0}(e^{qV_{CB}/kT} - 1) = A_1 e^{-W/L_B} + A_2 e^{W/L_B}$$

$$\Delta p_B(x) = \Delta p_B(0)\left(\frac{e^{(W-x)/L_B} - e^{-(W-x)/L_B}}{e^{W/L_B} - e^{-W/L_B}}\right) + \Delta p_B(W)\left(\frac{e^{x/L_B} - e^{-x/L_B}}{e^{W/L_B} - e^{-W/L_B}}\right)$$

$$\Delta p_B(x) = \Delta p_B(0)\frac{\sinh[(W-x)/L_B]}{\sinh(W/L_B)} + \Delta p_B(W)\frac{\sinh(x/L_B)}{\sinh(W/L_B)}$$

where $\sinh(\xi) \equiv \dfrac{e^{\xi} - e^{-\xi}}{2}$

Bipolar Junction Transistor (BJT) Quantitative Solution

Optional

The problem solution in the base quasi-neutral region gives us the hole currents...

$$I_{Ep} = -qAD_B \left.\frac{d\Delta p_B}{dx}\right|_{x=0}$$

$$I_{Cp} = -qAD_B \left.\frac{d\Delta p_B}{dx}\right|_{x=W}$$

$$I_{Ep} = qA\frac{D_B}{L_B}p_{B0}\left[\frac{\cosh(W/L_B)}{\sinh(W/L_B)}\left(e^{qV_{EB}/kT}-1\right)-\frac{1}{\sinh(W/L_B)}\left(e^{qV_{CB}/kT}-1\right)\right]$$

$$I_{Cp} = qA\frac{D_B}{L_B}p_{B0}\left[\frac{1}{\sinh(W/L_B)}\left(e^{qV_{EB}/kT}-1\right)-\frac{\cosh(W/L_B)}{\sinh(W/L_B)}\left(e^{qV_{CB}/kT}-1\right)\right]$$

where $\cosh(\xi) \equiv \frac{e^{\xi}+e^{-\xi}}{2}$

Bipolar Junction Transistor (BJT) Quantitative Solution

Adding it all together...

Optional

$$I_{En} = qA\frac{D_E}{L_E}n_{E0}(e^{qV_{EB}/kT}-1)$$

$$I_{Ep} = qA\frac{D_B}{L_B}p_{B0}\left[\frac{\cosh(W/L_B)}{\sinh(W/L_B)}(e^{qV_{EB}/kT}-1)-\frac{1}{\sinh(W/L_B)}(e^{qV_{CB}/kT}-1)\right]$$

$$I_{Cp} = qA\frac{D_B}{L_B}p_{B0}\left[\frac{1}{\sinh(W/L_B)}(e^{qV_{EB}/kT}-1)-\frac{\cosh(W/L_B)}{\sinh(W/L_B)}(e^{qV_{CB}/kT}-1)\right]$$

$$I_{Cn} = -qA\frac{D_C}{L_C}n_{C0}(e^{qV_{CB}/kT}-1)$$

Important Result from the Derivation

$$I_E = qA\left[\left(\frac{D_E}{L_E}n_{E0}+\frac{D_B}{L_B}p_{B0}\frac{\cosh(W/L_B)}{\sinh(W/L_B)}\right)(e^{qV_{EB}/kT}-1) - \left(\frac{D_B}{L_B}p_{B0}\frac{1}{\sinh(W/L_B)}\right)(e^{qV_{CB}/kT}-1)\right]$$

$$I_C = qA\left[\left(\frac{D_B}{L_B}p_{B0}\frac{1}{\sinh(W/L_B)}\right)(e^{qV_{EB}/kT}-1) - \left(\frac{D_C}{L_C}n_{C0}+\frac{D_B}{L_B}p_{B0}\frac{\cosh(W/L_B)}{\sinh(W/L_B)}\right)(e^{qV_{CB}/kT}-1)\right]$$

Bipolar Junction Transistor (BJT) Quantitative Solution

Optional

So the Performance Parameters are...

$$\gamma = \frac{1}{1 + \left(\frac{D_E}{D_B}\frac{L_B}{L_E}\frac{N_B}{N_E}\right)\frac{\sinh(W/L_B)}{\cosh(W/L_B)}}$$

$$\alpha_{dc} = \gamma\alpha_T = \frac{1}{\cosh(W/L_B) + \left(\frac{D_E}{D_B}\frac{L_B}{L_E}\frac{N_B}{N_E}\right)\sinh(W/L_B)}$$

$$\beta_{dc} = \frac{1}{\frac{1}{\alpha_{dc}} - 1} = \frac{1}{\cosh(W/L_B) + \left(\frac{D_E}{D_B}\frac{L_B}{L_E}\frac{N_B}{N_E}\right)\sinh(W/L_B) - 1}$$

Optional

When the base width is much less than the minority carrier diffusion length ...

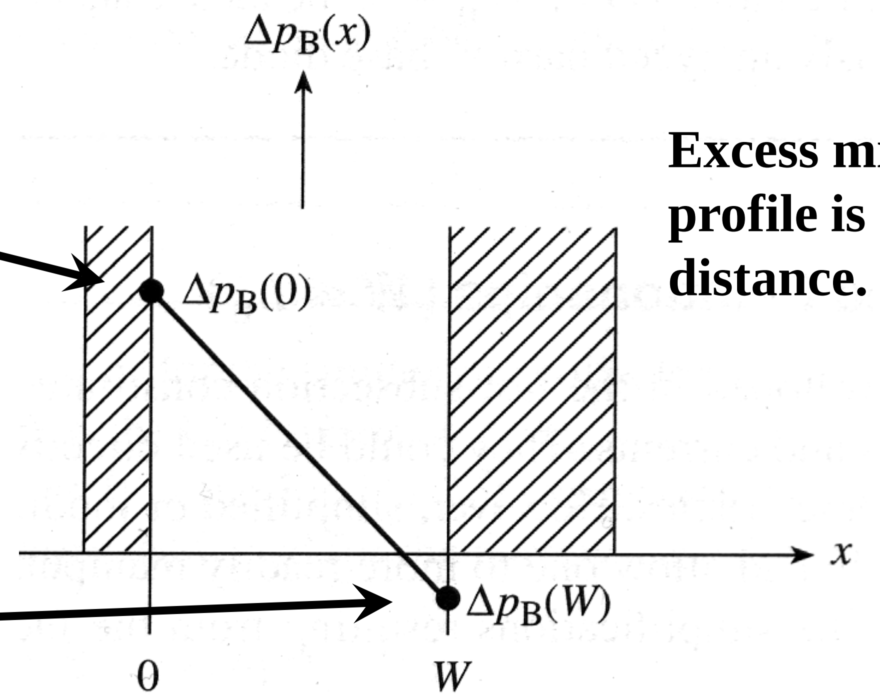
$$\sinh(\xi) \to \xi \qquad \ldots \xi \ll 1 \qquad \cosh(\xi) \to 1 + \frac{\xi^2}{2} \qquad \ldots \xi \ll 1$$

$$\Delta p_B(x) = \Delta p_B(0) + [\Delta p_B(W) - \Delta p_B(0)]\frac{x}{W}$$

In Active Mode

Excess minority carrier starts out positive due to forward biased emitter-base junction…

… and ends up negative due to reverse biased collector-base junction…



Excess minority carrier profile is linear with distance.

Bipolar Junction Transistor (BJT) Quantitative Solution

Insight into transistor performance

Optional

$$\gamma = \frac{1}{1 + \frac{D_E}{D_B}\frac{N_B}{N_E}\frac{W}{L_E}}$$

•Heavier emitter doping than base doping, $N_E >> N_B$, improves emitter efficiency (->1), dc common base current gain (->1), and dc common emitter current gain (larger)

$$\alpha_T = \frac{1}{1 + \frac{1}{2}\left(\frac{W}{L_B}\right)^2}$$

•A narrow base, $W << L_B$, improves base transport factor (->1)), dc common base current gain (->1), and dc common emitter current gain (larger)

$$\alpha_{dc} = \frac{1}{1 + \frac{D_E}{D_B}\frac{N_B}{N_E}\frac{W}{L_E} + \frac{1}{2}\left(\frac{W}{L_B}\right)^2}$$

$$\beta_{dc} = \frac{1}{\frac{D_E}{D_B}\frac{N_B}{N_E}\frac{W}{L_E} + \frac{1}{2}\left(\frac{W}{L_B}\right)^2}$$

Bipolar Junction Transistor (BJT) Quantitative Solution

Insight into transistor performance

Optional

If $L_B >> W$ (most of the minority carriers make it across the base),

$$\alpha_{DC} = \frac{1}{1 + \frac{D_E W N_B}{D_B L_E N_E} + \frac{1}{2}\left(\frac{W}{L_B}\right)^2} \Rightarrow \frac{1}{1 + \frac{D_E W N_B}{D_B L_E N_E}} = \frac{\beta_{DC}}{1 + \beta_{DC}}$$

and

$$\beta_{DC} = \frac{1}{\frac{D_E W N_B}{D_B L_E N_E} + \frac{1}{2}\left(\frac{W}{L_B}\right)^2} \Rightarrow \frac{D_B L_E N_E}{D_E W N_B} = \frac{\alpha_{DC}}{1 - \alpha_{DC}}$$