Lecture 18

Bipolar Junction Transistors (BJT): Part 2
Quantitative Understanding - Do the math!

Reading:
Pierret 11.1

Due to the shortened class period, we will not formally derive the Ebers Moll (Large Signal) model of the BJT. See the next lecture for a phenomenological development of the Ebers Moll Model.
Bipolar Junction Transistor (BJT) Quantitative Solution

The Definitions...

\[ N_E = N_{AE} \quad N_B = N_{DB} \quad N_C = N_{AC} \]
\[ D_E = D_N \quad D_B = D_P \quad D_C = D_N \]
\[ \tau_E = \tau_n \quad \tau_B = \tau_p \quad \tau_C = \tau_n \]
\[ L_E = L_N \quad L_B = L_P \quad L_C = L_N \]
\[ n_{E0} = n_{p0} = n_i^2/N_E \quad p_{B0} = p_{n0} = n_i^2/N_B \quad n_{C0} = n_{p0} = n_i^2/N_C \]
Bipolar Junction Transistor (BJT) Quantitative Solution

The Problem Setup...

Emitter Region
The diffusion equation to be solved is

$$0 = D_E \frac{d^2 \Delta n_E}{dx''^2} - \frac{\Delta n_E}{\tau_E}$$

subject to the boundary conditions

$$\Delta n_E(x'' \to \infty) = 0$$

$$\Delta n_E(x'' = 0) = n_{E0}(e^{qV_{EB}/kT} - 1)$$

Base Region
The diffusion equation to be solved is

$$0 = D_B \frac{d^2 \Delta p_B}{dx'^2} - \frac{\Delta p_B}{\tau_B}$$

subject to the boundary conditions

$$\Delta p_B(0) = p_{B0}(e^{qV_{EB}/kT} - 1)$$

$$\Delta p_B(W) = p_{B0}(e^{qV_{CB}/kT} - 1)$$

Collector Region
The diffusion equation to be solved is

$$0 = D_C \frac{d^2 \Delta n_C}{dx'^2} - \frac{\Delta n_C}{\tau_C}$$

subject to the boundary conditions

$$\Delta n_C(x' \to \infty) = 0$$

$$\Delta n_C(x' = 0) = n_{C0}(e^{qV_{CB}/kT} - 1)$$

Then Use

LAW OF THE JUNCTION!

From our diode boundary conditions

$$I_{En} = - qAD_E \frac{d\Delta n_E}{dx''} \bigg|_{x''=0}$$

$$I_{Ep} = - qAD_B \frac{d\Delta p_B}{dx} \bigg|_{x=0}$$

$$I_{Cp} = - qAD_B \frac{d\Delta p_B}{dx} \bigg|_{x=W}$$

$$I_{Cn} = qAD_C \frac{d\Delta n_C}{dx'} \bigg|_{x'=0}$$
Bipolar Junction Transistor (BJT) Quantitative Solution

The Problem Setup...

\[ I_{En} = qAD_E \frac{d\Delta n_E}{dx''} \bigg|_{x''=0} \]

\[ I_{Ep} = -qAD_B \frac{d\Delta p_B}{dx} \bigg|_{x=0} \]

\[ I_{Cp} = -qAD_B \frac{d\Delta p_B}{dx} \bigg|_{x=W} \]

\[ I_{Cn} = qAD_C \frac{d\Delta n_C}{dx'} \bigg|_{x'=0} \]

\[ \gamma = \frac{I_{Ep}}{I_{Ep} + I_{En}} \]

\[ \alpha_T = \frac{I_{Cp}}{I_{Ep}} \]

\[ \alpha_{dc} = \gamma \alpha_T \]

\[ \beta_{dc} = \frac{\alpha_{dc}}{1 - \alpha_{dc}} \]

\[ I_E = I_{Ep} + I_{En} \]

\[ I_C = I_{Cp} + I_{Cn} \]

\[ I_B = I_E - I_C \]
Bipolar Junction Transistor (BJT) Quantitative Solution

The Problem Setup...

Optional

Add electron and hole currents to obtain the total emitter current

Add electron and hole currents to obtain the total collector current

Find Hole Current here...

... will equal hole current here due to no recombination-generation in depletion regions

... add this current to the electron current here to determine the total current in the emitter and collector
Bipolar Junction Transistor (BJT) Quantitative Solution

The problem solution in the emitter and collector quasi-neutral regions...

\[ \Delta n_E(x'') = A_1 e^{-x''/L_E} + A_2 e^{x''/L_E} \]

\[ \Delta n_E(x'') = n_{E0} (e^{qV_{EB}/kT} - 1) e^{-x''/L_E} \]

\[ I_{En} = qA \frac{D_E}{L_E} n_{E0} (e^{qV_{EB}/kT} - 1) \]

\[ \Delta n_C(x') = n_{C0} (e^{qV_{CB}/kT} - 1) e^{-x'/L_C} \]

\[ I_{Cn} = -qA \frac{D_C}{L_C} n_{C0} (e^{qV_{CB}/kT} - 1) \]
Bipolar Junction Transistor (BJT) Quantitative Solution

The problem solution in the base quasi-neutral region...

\[ \Delta p_B(x) = A_1 e^{-x/L_B} + A_2 e^{x/L_B} \]

\[ \Delta p_B(0) = p_{B0}(e^{qV_{EB}/kT} - 1) = A_1 + A_2 \]

\[ \Delta p_B(W) = p_{B0}(e^{qV_{CB}/kT} - 1) = A_1 e^{-W/L_B} + A_2 e^{W/L_B} \]

\[ \Delta p_B(x) = \Delta p_B(0) \left( \frac{e^{(W-x)/L_B} - e^{-(W-x)/L_B}}{e^{W/L_B} - e^{-W/L_B}} \right) \]

\[ + \Delta p_B(W) \left( \frac{e^{x/L_B} - e^{-x/L_B}}{e^{W/L_B} - e^{-W/L_B}} \right) \]

\[ \Delta p_B(x) = \Delta p_B(0) \frac{\sinh[(W - x)/L_B]}{\sinh(W/L_B)} + \Delta p_B(W) \frac{\sinh(x/L_B)}{\sinh(W/L_B)} \]

where \( \sinh(\xi) \equiv \frac{e^{\xi} - e^{-\xi}}{2} \)
Bipolar Junction Transistor (BJT) Quantitative Solution

The problem solution in the base quasi-neutral region gives us the hole currents...

\[ I_{Ep} = -qAD_B \frac{d\Delta p_B}{dx} \bigg|_{x=0} \]

\[ I_{Cp} = -qAD_B \frac{d\Delta p_B}{dx} \bigg|_{x=W} \]

\[ I_{Ep} = qA \frac{D_B}{L_B} p_{B0} \left[ \frac{\cosh(W/L_B)}{\sinh(W/L_B)} \left( e^{qV_{EB}/kT} - 1 \right) - \frac{1}{\sinh(W/L_B)} \left( e^{qV_{CB}/kT} - 1 \right) \right] \]

\[ I_{Cp} = qA \frac{D_B}{L_B} p_{B0} \left[ \frac{1}{\sinh(W/L_B)} \left( e^{qV_{EB}/kT} - 1 \right) - \frac{\cosh(W/L_B)}{\sinh(W/L_B)} \left( e^{qV_{CB}/kT} - 1 \right) \right] \]

where \( \cosh(\xi) = \frac{e^\xi + e^{-\xi}}{2} \)
Bipolar Junction Transistor (BJT) Quantitative Solution

Adding it all together...

\[ I_{E0} = \frac{kT}{nE0} \left( e^{qV_{EB}/kT} - 1 \right) \]

\[ I_{Ep} = qA \frac{D_B}{L_B} p_{BO} \left[ \frac{\cosh(W/L_B)}{\sinh(W/L_B)} \left( e^{qV_{EB}/kT} - 1 \right) - \frac{1}{\sinh(W/L_B)} \left( e^{qV_{CB}/kT} - 1 \right) \right] \]

\[ I_{Cp} = qA \frac{D_B}{L_B} p_{BO} \left[ \frac{1}{\sinh(W/L_B)} \left( e^{qV_{EB}/kT} - 1 \right) - \frac{\cosh(W/L_B)}{\sinh(W/L_B)} \left( e^{qV_{CB}/kT} - 1 \right) \right] \]

\[ I_{Cn} = -qA \frac{D_C}{L_C} n_{C0} \left( e^{qV_{CB}/kT} - 1 \right) \]

Important Result from the Derivation

\[ I_E = qA \left[ \left( \frac{D_E}{L_E} \right) n_{E0} + \frac{D_B}{L_B} p_{BO} \frac{\cosh(W/L_B)}{\sinh(W/L_B)} \left( e^{qV_{EB}/kT} - 1 \right) \right] \]

\[ - \left( \frac{D_B}{L_B} p_{BO} \frac{1}{\sinh(W/L_B)} \right) \left( e^{qV_{CB}/kT} - 1 \right) \]

\[ I_C = qA \left[ \left( \frac{D_B}{L_B} p_{BO} \frac{1}{\sinh(W/L_B)} \right) \left( e^{qV_{EB}/kT} - 1 \right) \right] \]

\[ - \left( \frac{D_C}{L_C} n_{C0} + \frac{D_B}{L_B} p_{BO} \frac{\cosh(W/L_B)}{\sinh(W/L_B)} \right) \left( e^{qV_{CB}/kT} - 1 \right) \]
Bipolar Junction Transistor (BJT) Quantitative Solution

So the Performance Parameters are...

\[ \gamma = \frac{1}{1 + \left( \frac{D_E}{D_B} \frac{L_B}{L_E} \frac{N_B}{N_E} \right) \frac{\sinh(W/L_B)}{\cosh(W/L_B)}} \]

\[ \alpha_{dc} = \gamma \alpha_T = \frac{1}{\cosh(W/L_B) + \left( \frac{D_E}{D_B} \frac{L_B}{L_E} \frac{N_B}{N_E} \right) \sinh(W/L_B)} \]

\[ \beta_{dc} = \frac{1}{1 - \frac{1}{\alpha_{dc}}} = \frac{1}{\cosh(W/L_B) + \left( \frac{D_E}{D_B} \frac{L_B}{L_E} \frac{N_B}{N_E} \right) \sinh(W/L_B) - 1} \]
Bipolar Junction Transistor (BJT) Quantitative Solution

When the base width is much less than the minority carrier diffusion length ...

\[ \sinh(\xi) \rightarrow \xi \quad \ldots \quad \xi \ll 1 \quad \cosh(\xi) \rightarrow 1 + \frac{\xi^2}{2} \quad \ldots \quad \xi \ll 1 \]

\[ \Delta p_B(x) = \Delta p_B(0) + \left[ \Delta p_B(W) - \Delta p_B(0) \right] \frac{x}{W} \]

**In Active Mode**
Excess minority carrier starts out positive due to forward biased emitter-base junction...

... and ends up negative due to reverse biased collector-base junction...

Excess minority carrier profile is linear with distance.
Bipolar Junction Transistor (BJT) Quantitative Solution

Insight into transistor performance

\[ \gamma = \frac{1}{1 + \frac{D_E}{D_B} \frac{N_B}{N_E} \frac{W}{L_E}} \]

- Heavier emitter doping than base doping, \( N_E \gg N_B \), improves emitter efficiency (\( \rightarrow 1 \)), dc common base current gain (\( \rightarrow 1 \)), and dc common emitter current gain (larger)

\[ \alpha_T = \frac{1}{1 + \frac{1}{2} \left( \frac{W}{L_B} \right)^2} \]

- A narrow base, \( W \ll L_B \), improves base transport factor (\( \rightarrow 1 \)), dc common base current gain (\( \rightarrow 1 \)), and dc common emitter current gain (larger)

\[ \alpha_{dc} = \frac{1}{1 + \frac{D_E}{D_B} \frac{N_B}{N_E} \frac{W}{L_E} + \frac{1}{2} \left( \frac{W}{L_B} \right)^2} \]

\[ \beta_{dc} = \frac{1}{\frac{D_E}{D_B} \frac{N_B}{N_E} \frac{W}{L_E} + \frac{1}{2} \left( \frac{W}{L_B} \right)^2} \]
Bipolar Junction Transistor (BJT) Quantitative Solution

Insight into transistor performance

If $L_B \gg W$ (most of the minority carriers make it across the base),

$$\alpha_{DC} = \frac{1}{1 + \frac{D_E W N_B}{D_B L_E N_E} + \frac{1}{2} \left( \frac{W}{L_B} \right)^2} \quad \Rightarrow \quad \frac{1}{1 + \frac{D_E W N_B}{D_B L_E N_E}} = \frac{\beta_{DC}}{1 + \beta_{DC}}$$

and

$$\beta_{DC} = \frac{1}{\frac{D_E W N_B}{D_B L_E N_E} + \frac{1}{2} \left( \frac{W}{L_B} \right)^2} \Rightarrow \frac{D_B L_E N_E}{D_E W N_B} = \frac{\alpha_{DC}}{1 - \alpha_{DC}}$$