Lecture 19

Bipolar Junction Transistors (BJT): Part 3

Ebers Moll Large Signal BJT Model, Using CVD model to solve for DC bias point

Reading:

Pierret 11.1
Bipolar Junction Transistor (BJT) Quantitative Solution

Insight into transistor performance

If \( L_B \gg W \) (most of the minority carriers make it across the base),

\[
\alpha_{DC} = \frac{1}{1 + \frac{D_E W N_B}{D_B L_E N_E} + \frac{1}{2} \left( \frac{W}{L_B} \right)^2} \quad \Rightarrow \quad \frac{1}{1 + \frac{D_E W N_B}{D_B L_E N_E}} = \frac{\beta_{DC}}{1 + \beta_{DC}}
\]

and

\[
\beta_{DC} = \frac{1}{\frac{D_E W N_B}{D_B L_E N_E} + \frac{1}{2} \left( \frac{W}{L_B} \right)^2} \quad \Rightarrow \quad \frac{D_B L_E N_E}{D_E W N_B} = \frac{\alpha_{DC}}{1 - \alpha_{DC}}
\]
Development of the Large Signal Model of a BJT (Ebers-Moll Model)

\[
I_E = qA \left( \frac{D_E}{L_E} n_{Eo} + \frac{D_B P_{Bo}}{L_B} \cosh \left( \frac{W}{L_B} \right) \right) \left( e^{V_{EB}/V_T} - 1 \right) - qA \left( \frac{D_B P_{Bo}}{L_B} \frac{1}{\sinh \left( \frac{W}{L_B} \right)} \right) \left( e^{V_{CB}/V_T} - 1 \right)
\]

\[
I_C = qA \left( \frac{D_B P_{Bo}}{L_B} \frac{1}{\sinh \left( \frac{W}{L_B} \right)} \right) \left( e^{V_{EB}/V_T} - 1 \right) - qA \left( \frac{D_C}{L_C} n_{Co} + \frac{D_B P_{Bo}}{L_B} \cosh \left( \frac{W}{L_B} \right) \right) \left( e^{V_{CB}/V_T} - 1 \right)
\]

\[
I_E = I_{F0} \left( e^{V_{EB}/V_T} - 1 \right) - A \left( e^{V_{CB}/V_T} - 1 \right)
\]

\[
I_C = A \left( e^{V_{EB}/V_T} - 1 \right) - I_{R0} \left( e^{V_{CB}/V_T} - 1 \right)
\]

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\[
I_E = I_{F0} \left( e^{\frac{V_{EB}}{V_T}} - 1 \right) - A \left( e^{\frac{V_{CB}}{V_T}} - 1 \right)
\]

\[
I_C = A \left( e^{\frac{V_{EB}}{V_T}} - 1 \right) - I_{R0} \left( e^{\frac{V_{CB}}{V_T}} - 1 \right)
\]

When \( V_{CB} = 0 \),

\[
I_E = I_{F0} \left( e^{\frac{V_{EB}}{V_T}} - 1 \right) \quad \text{and} \quad I_C = A \left( e^{\frac{V_{EB}}{V_T}} - 1 \right)
\]

but,

\( I_{F0} > A \) \ (see *)

Thus,

\[
I_E = I_{F0} \left( e^{\frac{V_{EB}}{V_T}} - 1 \right) \quad \text{and} \quad I_C = \alpha_F I_{F0} \left( e^{\frac{V_{EB}}{V_T}} - 1 \right)
\]

but, \( I_C = \alpha_F I_E \rightarrow \alpha_F = \alpha_{DC} \) \ common base current gain

The collector current is the fraction of the emitter current “collected”
Development of the Large Signal Model of a BJT (Ebers-Moll Model)

\[ I_E = I_{F0} \left( e^{\frac{V_{EB}}{V_T}} - 1 \right) - A \left( e^{\frac{V_{CB}}{V_T}} - 1 \right) \]

\[ I_C = A \left( e^{\frac{V_{EB}}{V_T}} - 1 \right) - I_{R0} \left( e^{\frac{V_{CB}}{V_T}} - 1 \right) \]

When \( V_{EB} = 0 \),

\[ I_E = -A \left( e^{\frac{V_{CB}}{V_T}} - 1 \right) \quad \text{and} \quad I_C = -I_{R0} \left( e^{\frac{V_{CB}}{V_T}} - 1 \right) \]

but,

\[ I_{R0} > A \quad (\text{see} \, *) \]

Thus,

\[ I_E = -\alpha_R I_{R0} \left( e^{\frac{V_{CB}}{V_T}} - 1 \right) \quad \text{and} \quad I_C = -I_{R0} \left( e^{\frac{V_{CB}}{V_T}} - 1 \right) \]

but, \( I_E = \alpha_R I_C \rightarrow \alpha_R \neq \alpha_{DC} \)

In Inverse Active mode, the emitter current is the fraction of the collector current “collected”
Development of the Large Signal Model of a BJT (Ebers-Moll Model)

Note: \( A = \alpha_R I_{R0} = \alpha_F I_{F0} \)

PNP

\[
I_F = I_{F0} \left( e^{V_{EB}/V_T} - 1 \right) \quad \text{and} \quad I_R = I_{R0} \left( e^{V_{CB}/V_T} - 1 \right)
\]

Ideal Diodes

\[
I_E = I_{F0} \left( e^{V_{EB}/V_T} - 1 \right) - \alpha_R I_{R0} \left( e^{V_{CB}/V_T} - 1 \right)
\]

\[
I_C = \alpha_F I_{F0} \left( e^{V_{EB}/V_T} - 1 \right) - I_{R0} \left( e^{V_{CB}/V_T} - 1 \right)
\]
Development of the Large Signal Model of a BJT (Ebers-Moll Model)

**NPN**

\[ I_F = I_{F0} \left( e^{V_{BE}/V_T} - 1 \right) \]

**Ideal Diodes**

\[ I_R = I_{R0} \left( e^{V_{BC}/V_T} - 1 \right) \]

**Diodes Ideal**

\[ V_{BE} \]

\[ V_{BC} \]

\[ V_{BB} \]

\[ V_{BB} \]

\[ I_E = I_{F0} \left( e^{V_{BE}/V_T} - 1 \right) - \alpha_R I_{R0} \left( e^{V_{BC}/V_T} - 1 \right) \]

\[ I_C = \alpha_F I_{F0} \left( e^{V_{BE}/V_T} - 1 \right) - I_{R0} \left( e^{V_{BC}/V_T} - 1 \right) \]
Using the Ebers-Moll model requires mathematical complexity (and much pain). Thus, we have an approximate solution method* that allows a quick solution.

*I refer to as the “CVD/Beta Analysis”. This is just my term, not a universal name.
Quick Solution using a CVD/Beta Approach

Consider the following pnp BJT circuit with a common emitter current gain, $\beta_{dc}=180.7$. Find $I_b$, $I_c$, and $I_e$ assuming a turn on voltage of 0.7V.

Neglect Leakage currents

\[
I_c = \alpha_{dc} I_e + I_{CB0}
\]
\[
I_c = \beta_{dc} I_b + I_{CEo}
\]
\[
I_e = I_b + I_c
\]

\[
0 = -4V + I_B(12000) + V_{EB} + I_E(15000)
\]
\[
4V = I_B(12000) + 0.7V + I_C(1/\alpha_{DC})(15000)
\]
\[
4V = I_B(12000) + 0.7V + [\beta_{DC}I_B][(1+\beta_{DC})/\beta_{DC}](15000)
\]
\[
3.3V = I_B[(12000)+(1+180.7)(15000)]
\]

\[
I_B = 1.2\mu A \\
I_C = 180.7I_B = 218\mu A \\
I_E = (181.7/180.7)I_C = 219\mu A
\]
Development of the Large Signal Model of a BJT (Ebers-Moll Model)

Compare our results using the CVD/Beta model to the full Ebers-Moll solution used in PSPICE...

Actual $V_{be} = 0.662\text{V}$ not 0.7V as assumed

Actual $I_{base} = 1.05\text{uA}$ not 1.2uA as calculated

Only 1% error in the collector and emitter currents

Current into various nodes

Voltage at various nodes
Development of the Large Signal Model of a BJT (Ebers-Moll Model)

Common Base

IV curve looks like a diode

Real shows variation due to “base width modulation” dependent on the applied $V_{CB}$

After the base-collector junction is reverse biased (starts collecting), $I_E \approx I_C$

Real IV is limited by breakdown of the base-collector junction

Input Output

$I_E$ and $I_C$ and $V_{EB}$ and $-V_{CB}$

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Common Emitter

IV curve looks like a diode but has a DC shift associated with the reverse biased base-collector junction current.

After the base-collector junction is reverse biased (starts collecting), $I_C = \beta I_B$.

Real IV is limited by breakdown of the base-collector junction.

Real shows finite slope due to "base width modulation" dependent on the applied $V_{CB}$.