Lecture 27

Amplifier Configurations

Reading:

CE/CS: Jaeger 13.6, 13.9, 13.10, 13.11

CC/CD: Jaeger 14.1, 14.3

CB/CG: Jaeger 14.1, 14.4

and Notes
Amplifier Configurations

Voltage Amplifier: Voltage input and Voltage output

- Any signal source has a finite “source resistance”, $R_S$.
- The amplifier is often asked to drive current into a load of finite impedance, $R_L$ (examples: 8 ohm speaker, 50 ohm transmission line, etc…)

The controlled source is a Voltage-controlled-Voltage Source

$A_v=$ Open Circuit Voltage Gain can be found by applying a voltage source with $R_s=0$, and measuring the open circuit output voltage (no load or $R_L=\infty$)
Amplifier Configurations

Why is the input and output resistance important?

- Only the voltage $v_{in}$ is amplified to $A_v v_{in}$.
- Since $R_s$ and $R_{in}$ form a voltage divider that determines $v_{in}$, you want $R_{in}$ as large as possible (for a voltage amplifier) for maximum voltage gain.
- Since $R_L$ and $R_{out}$ form a voltage divider that determines $v_{out}$, you want $R_{out}$ as small as possible (for a voltage amplifier) for maximum voltage gain.
Amplifier Configurations

Current Amplifier: Current input and Current output

The controlled source is a Current-controlled-Current Source

$A_i=$Short Circuit Current Gain can be found by applying a current source with $R_s=\infty$, and measuring the short circuit output current (No Load or $R_L=0$)

- Only the current $i_{in}$ is amplified to $A_i i_{in}$.
- Since $R_s$ and $R_{in}$ form a current divider that determines $i_{in}$, you want $R_{in}$ as small as possible (for a current amplifier) for maximum current gain.
- Since $R_L$ and $R_{out}$ form a current divider that determines $i_{out}$, you want $R_{out}$ as large as possible (for a current amplifier) for maximum current gain.
Amplifier Configurations

Transconductance Amplifier: Voltage input and Current output

The controlled source is a Voltage-controlled-Current Source

\[ G_m = \text{Transconductance Gain} \]

can be found by applying a voltage source with \( R_s = 0 \), and measuring the short circuit output current (No Load or \( R_L = 0 \))

• Only the voltage \( v_{in} \) is amplified to \( i_{out} = G_m v_{in} \).
• Since \( R_s \) and \( R_{in} \) form a voltage divider that determines \( v_{in} \), you want \( R_{in} \) as large as possible for maximum transconductance gain.
• Since \( R_L \) and \( R_{out} \) form a current divider that determines \( i_{out} \), you want \( R_{out} \) as large as possible for maximum transconductance gain.
Amplifier Configurations

Transresistance Amplifier: Current input and Voltage output

The controlled source is a Current-controlled-Voltage Source

\[ R_m = \text{Transresistance Gain} \]

can be found by applying a current source with \( R_s = \text{infinity} \), and measuring the open circuit output voltage (\( R_L = \text{infinity} \))

- Only the current \( i_{in} \) is amplified to \( v_{out} = R_m i_{in} \)
- Since \( R_s \) and \( R_{in} \) form a current divider that determines \( i_{in} \), you want \( R_{in} \) as small as possible for maximum transresistance gain.
- Since \( R_L \) and \( R_{out} \) form a voltage divider that determines \( v_{out} \), you want \( R_{out} \) as small as possible for maximum transresistance gain.
Amplifier Configurations

Input Resistance
With the load resistance attached…
Apply a test input voltage and measure the input current, \( R_{\text{in}} = \frac{v_t}{i_t} \)
Or
Apply a test input current and measure the input voltage, \( R_{\text{in}} = \frac{v_t}{i_t} \)

Output Resistance
With all input voltage sources shorted and all input current sources opened…
Apply a test voltage to the output and measure the output current, \( R_{\text{out}} = \frac{v_t}{i_t} \)
Or
Apply a test current to the output and measure the output voltage, \( R_{\text{out}} = \frac{v_t}{i_t} \)
Final Summary of Transistor Amplifier Analysis

1.) a.) Determine DC operating point. Make sure the transistors are biased into active mode (forward active for BJTs and Saturation for MOSFET. Do not confuse the two terms as saturation means a completely different thing for a BJT) and b.) calculate small signal parameters $g_m$, $r_i$, $r_o$ etc…

2.) Convert to the AC only model.
   • DC Voltage sources are replaced with shorts to ground
   • DC Current sources are replaced with open circuits
   • Large capacitors are replaced with short circuits
   • Large inductors are replaced with open circuits

3.) Use a Thevenin circuit where necessary on each leg of transistor

4.) Replace transistor with small signal model

5.) Simplify the circuit as much as necessary and solve for gain.

6.) Solve for Input Resistance: With the load resistance attached… a.) Apply a test input voltage and measure the input current, $R_{in} = v_t/i_t$ or b.) Apply a test input current and measure the input voltage, $R_{in} = v_t/i_t$

7.) Solve for Output Resistance: With all input voltage sources shorted and all input current sources opened… a.) Apply a test voltage to the output and measure the output current, $R_{out} = v_t/i_t$ or b.) Apply a test current to the output and measure the output voltage, $R_{out} = v_t/i_t$
Transistor Amplifier Configurations

Common Emitter and Common Source

Can be modeled as a current amplifier, $I_C = \beta I_B$, or a transconductance amplifier, $i_C = K v_{BE}$

Modeled as transconductance amplifier, $i_{DS} = K v_{GS}$

Overall Amplifier Configuration

- Emitter/Source is neither an input nor an output
- Input is between base-emitter or gate-source
- Output is between collector-emitter and drain-source
- Is a transconductance amplifier (see small signal models we have used in previous examples)
Previously, we have analyzed voltage gain. Now let us look at the amplifier input and output resistance (these are small signal parameters):

\[ Rin = R2 \parallel R1 \parallel r_\pi \text{ for the BJT} \quad \text{or} \quad Rin = R2 \parallel R1 \text{ for the MOSFET} \]

\[ Rout = r_o \parallel R_c \text{ for the BJT or MOSFET} \]
Transistor Amplifier Configurations

Summary of Common Emitter and Common Source Characteristics

• Very Large Voltage Gain
• Inverting Voltage Gain (due to $-g_m r_o$)
• High Input Impedance
• High Output Impedance

These properties make the CE/CS configuration very good for high gain stages of amplifiers.

Now let us consider the other two configurations of transistor amplifiers:

• Common Gate/Common Base
• Common Drain/Common Collector
Transistor Amplifier Configurations
Common Collector and Common Drain
DC Circuit

Collector (or Drain) is neither an input or output
Input is Base (or Gate)
Output is Emitter (or Source)
Transistor Amplifier Configurations

Common Collector

DC Circuit converted to AC Equivalent (reduced)

DC Circuit

Note the extra ground due to $C_2$

AC Circuit

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Transistor Amplifier Configurations

Common Collector

DC Circuit converted to AC Equivalent (reduced)

AC Circuit

\[ v_o = (\beta_o + 1)i_b \left( R_4 \parallel r_o \parallel R_7 \right) \]

\[ v_{th} = i_b \left( R_{th} + r_\pi + (\beta_o + 1) \left( R_4 \parallel r_o \parallel R_7 \right) \right) \]
Transistor Amplifier Configurations

Common Collector

AC Voltage Gain

\[ v_o = (\beta_o + 1)i_b(R4 \parallel r_o \parallel R7) \]

\[ v_{ih} = i_b(R_{th} + r_\pi + (\beta_o + 1)(R4 \parallel r_o \parallel R7)) \]

\[ v_{ih} = v_i \frac{R2 \parallel R1}{R2 \parallel R1 + R_s} \]

\[ A_v = \frac{v_o}{v_{ih}} \frac{v_{ih}}{v_s} = \left( \frac{R2 \parallel R1}{R2 \parallel R1 + R_s} \right) \frac{(\beta_o + 1)i_b(R4 \parallel r_o \parallel R7)}{i_b(R_{th} + r_\pi + (\beta_o + 1)(R4 \parallel r_o \parallel R7))} \]

\[ A_v = \left( \frac{R2 \parallel R1}{R2 \parallel R1 + R_s} \right) \frac{(\beta_o + 1)R_L}{(R_{th} + r_\pi + (\beta_o + 1)R_L)} \text{ where } R_L = (R4 \parallel r_o \parallel R7) \]

Multiplying numerator and denominator by \( \frac{g_m}{(\beta_o + 1)} \)

\[ A_v = \left( \frac{R2 \parallel R1}{R2 \parallel R1 + R_s} \right) \left( \frac{g_mR_L}{g_m\left(\frac{R_{th}}{(\beta_o + 1)} + R_L\right) + g_m r_\pi \left(\frac{1}{(\beta_o + 1)}\right)} \right) \]

\[ A_v = \left( \frac{R2 \parallel R1}{R2 \parallel R1 + R_s} \right) \left( \frac{g_mR_L}{g_m\left(\frac{R_{th}}{(\beta_o + 1)} + R_L\right) + \alpha_o} \right) \]

But for \( (R2 \parallel R1) >> R_s \) and \( g_mR_L >> 1 \)

\[ A_v \approx \frac{g_mR_L}{1 + g_mR_L} \approx 1 \left[ \frac{V_o}{V_i} \right] \]

Gain is positive and \(~1\)
Transistor Amplifier Configurations
Common Drain Conversion from DC to AC Equivalent Circuit

DC Circuit

AC Circuit
Transistor Amplifier Configurations

Common Emitter and Common Source

DC Circuit converted to AC Equivalent (reduced)

AC Circuit

AC Circuit (reduced)
Transistor Amplifier Configurations
Common Drain AC Voltage Gain

\[ v_o = g_m v_{GS} (R4 \parallel r_o \parallel R7) \]
\[ v_{th} = v_{GS} + g_m v_{GS} (R4 \parallel r_o \parallel R7) = v_{GS} (1 + g_m (R4 \parallel r_o \parallel R7)) \]
\[ v_{th} = v_s \frac{R2 \parallel R1}{R2 \parallel R1 + R_s} \]
\[ A_v = \frac{v_o}{v_{GS}} \frac{v_{th}}{v_s} = \left( \frac{R2 \parallel R1}{R2 \parallel R1 + R_s} \right) \left( \frac{g_m (R4 \parallel r_o \parallel R7)}{(1 + g_m (R4 \parallel r_o \parallel R7))} \right) \]
\[ A_v = \left( \frac{R2 \parallel R1}{R2 \parallel R1 + R_s} \right) \left( \frac{g_m R_L}{(1 + g_m R_L)} \right) \text{where } R_L = (R4 \parallel r_o \parallel R7) \]

But for \((R2 \parallel R1) \gg R_s \text{ and } g_m R_L \gg 1\)
\[ A_v \approx \frac{g_m R_L}{1 + g_m R_L} \approx 1 \left[ \frac{V}{V} \right] \]
Gain is positive and \(~1\)
Transistor Amplifier Configurations
Common Collector/Drain Input Resistance

Input Resistance: With the load resistance attached apply a test input voltage and measure the input current, \( R_{in} = \frac{v_x}{i} \) (where \( i = i_x \))

\[
R_{in,BJT} = \frac{v_x}{i} = r_\pi + (\beta_o + 1)R_L
\]

Resistance in the emitter circuit is “multiplied” by transistor to increase the input resistance

\[
R_{in,MOSFET} = \frac{v_x}{i} = \frac{v_x}{0} = \infty
\]
Transistor Amplifier Configurations
Common Collector Output Resistance

Output Resistance: With all input voltage sources shorted and all input current sources opened, apply a test voltage to the output and measure the output current, \( R_{out} = \frac{v_x}{i_x} \)

\[
i_x = -i - \beta_o i + \frac{v_x}{R_L} = \frac{v_x}{r_\pi + R_{th}} - \beta_o i + \frac{v_x}{r_o} = \frac{v_x}{r_\pi + R_{th}} - \beta_o \left( -\frac{v_x}{r_\pi + R_{th}} \right) + \frac{v_x}{R_L}
\]

\[
R_{out,BJT} = \frac{v_x}{i_x} = \frac{1}{\frac{1}{r_\pi + R_{th}} + \frac{1}{R_L}} \left( \frac{1}{\beta_o + 1} \right)
\]

where \( R_L = r_o \parallel R4 \)

Two resistors in parallel: \( R_L \), and Resistance in the base circuit is “multiplied” by transistor to decrease the output resistance
Output Resistance: With all input voltage sources shorted and all input current sources opened, apply a test voltage to the output and measure the output current, \( R_{out} = \frac{v_x}{i} \)

\[
i_x = -g_m v_{GS} + \frac{v_x}{r_o} = g_m v_x + \frac{v_x}{R_L}
\]

\[
R_{out,MOSFET} = \frac{v_x}{i_x} = \frac{1}{g_m + \frac{1}{R_L}} \quad \text{where } R_L = r_o \parallel R4
\]

**Two resistors in parallel:** \( R_L \) and inverse transconductance
Transistor Amplifier Configurations

Summary of Common Collector and Common Drain Characteristics

- Unity Voltage Gain
- **Non-Inverting** Voltage Gain
- Very High Input Impedance
- Low Output Impedance

These properties make the CC/CD configuration very good for impedance transformation, I.E. “buffering” high impedances to low impedances. CC/CD configurations are good for output stages of amplifiers due to their very low output impedance, I.E., very little voltage drop in the output resistance of the amp.
Transistor Amplifier Configurations

Common Collector/Drain Current Gain

\[ A_{i,BJT} = \frac{i}{i_{th}} = \beta_o + 1 \]

\[ A_{i,MOSFET} = \frac{i}{i_{th}} = \infty \]

Note: since R7 was originally defined as the load, the current gain should actually be \((\beta+1) \frac{(R4||r_o)}{(R4||r_o+R7)}\) using a current divider.
Transistor Amplifier Configurations
Common Base and Common Gate
DC Circuit

Base (or Gate) is neither an input or output
Input is Emitter (or Source)
Output is Collector (or Drain)
Transistor Amplifier Configurations

Common Base

DC Circuit converted to AC Equivalent (reduced)

DC Circuit

Note: Jaeger let’s \( r_o \) go to infinity which makes the math dramatically easier

AC Circuit
Transistor Amplifier Configurations

Common Base AC Equivalent (reduced)

AC Circuit

\[ V_{th} = V_s \frac{R_4}{R_4 + R_s} = 0.8667V_s \]

\[ R_{th} = R_s \parallel R_4 = 1.73K \]
Transistor Amplifier Configurations

Common Base Voltage Gain

\[ i_E = i_C + i_B = g_m v_\pi + i + i_B \]

\[ i_E = g_m v_\pi + \left( \frac{v_\pi + v_o}{r_o} \right) + \left( \frac{v_\pi}{r_\pi} \right) \]

\[ v_{ih} = -i_E R_{th} - i_B r_\pi \]

\[-v_{th} = \left( g_m v_\pi + \left( \frac{v_\pi + v_o}{r_o} \right) + \left( \frac{v_\pi}{r_\pi} \right) \right) R_{th} + \left( \frac{v_\pi}{r_\pi} \right) r_\pi \]

also,

\[ v_o = -i_C R_L = \left( g_m v_\pi + \left( \frac{v_\pi + v_o}{r_o} \right) \right) R_L \]

so,

\[ v_o = -v_\pi \left( g_m + \frac{1}{r_o} \right) \left( \frac{R_L}{1 + \frac{R_L}{r_o}} \right) \]

\[ -v_{th} = \left( g_m v_\pi + \left( \frac{v_\pi - v_o}{r_o} \right) \frac{R_L}{1 + \frac{R_L}{r_o}} \right) + \left( \frac{v_\pi}{r_\pi} \right) R_{th} + v_\pi \]

\[ V_{th} = v_\pi \frac{R4}{R4 + R_s} = 0.8667 V_s \]

\[ R_{th} = R_s || R4 = 1.73 K \]
Transistor Amplifier Configurations

Common Base Voltage Gain

\[-v_{th} = g_m v_x + \left( v_x - v_{th} \right) \frac{g_m + \frac{1}{r_o}}{R_L}\left( \frac{1 + R_L}{r_o} \right) R_{th} + v_x \Rightarrow -v_{th} = v_x\]

\[A_v = \frac{v_o}{v_x} \frac{v_{th}}{v_s} = -\left( \frac{g_m + \frac{1}{r_o}}{1 + \frac{R_L}{r_o}} \right) R_L \frac{-1}{R_4} \left( \frac{R_4}{R_4 + R_s} \right) \]

\[g_m + \left( \frac{1 + \frac{R_L}{r_o}}{1 + \frac{R_L}{r_o}} \right) R_{th} + \left( \frac{1}{r_{th}} \right) R_{th} + 1\]

\[for \ r_o \rightarrow \infty \ and \ R_4 \gg R_s\]

\[A_v = \frac{v_o}{v_x} \frac{v_{th}}{v_s} = \frac{g_m R_L}{g_m R_{th} + \left( \frac{R_{th}}{r_{th}} \right) + 1}\]

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Transistor Amplifier Configurations

Common Base Voltage Gain

\[ A_v = \frac{g_m R_L}{g_m R_{th} + \left( \frac{R_{th}}{r_\pi} \right) + 1} \]

\[ A_v = \frac{g_m R_L}{R_{th} \left( g_m + \frac{1}{r_\pi} \right) + 1} \]

\[ A_v = \frac{g_m R_L}{R_{th} \left( g_m \frac{g_m r_\pi + 1}{g_m r_\pi} \right) + 1} \]

\[ A_v = \frac{g_m R_L}{R_{th} \left( \frac{g_m}{g_m r_\pi + 1} \right) + 1} \]

\[ A_v = \frac{g_m R_L}{R_{th} \left( \frac{g_m}{\beta_0 + 1} \right) + 1} \quad \Rightarrow \quad \text{Large } \beta_0 \]

\[ A_v = \frac{g_m R_L}{1 + R_{th} g_m} \]

Gain is positive and can be large
Transistor Amplifier Configurations

Common Base Input Resistance

\[ r_o = -v_x \]

\[ i_c = i_B \]

\[ v_x = -v_{\pi} \]

**Input Resistance:** With the load resistance attached apply a test input voltage and measure the input current, \( R_{\text{in}} = v_x / i \)

*From before,*

\[ i_E = g_m v_{\pi} + \left( \frac{v_{\pi}}{r_o} \right) + \left( \frac{v_{\pi}}{r_{\pi}} \right) \]

\[ v_o = -v_{\pi} \left( g_m + \frac{1}{r_o} \right) R_L \]

\[ i_x = -i_E \quad \text{and} \quad v_x = -v_{\pi} \]

\[ i_x = g_m v_x + \left( \frac{v_x - v_o}{r_o} \right) + \left( \frac{v_x}{r_{\pi}} \right) \]

\[ v_o = v_x \left( g_m + \frac{1}{r_o} \right) \frac{1}{R_L + \frac{R_i}{r_o}} R_L \]
Transistor Amplifier Configurations

Common Base Input Resistance

\[ i_x = g_m v_x + \frac{v_x - v_i}{r_o} \left( \frac{g_m + \frac{1}{r_o}}{1 + \frac{R_L}{r_o}} \right) R_L + \frac{v_x}{r_o} \]

\[ i_x = v_x g_m + \frac{1}{r_o} \left( 1 - \frac{g_m + \frac{1}{r_o}}{1 + \frac{R_L}{r_o}} R_L \right) R_L + \frac{1}{r_o} \]

\[ R_{in,BJT} = \frac{v_x}{i_x} = \frac{1}{g_m + \frac{1}{r_o} + \frac{1}{r_\pi}} \]

Letting \( r_o \to \infty \)

\[ R_{in,BJT} = \frac{v_x}{i_x} = \frac{1}{g_m + \frac{1}{r_\pi}} \]

\[ R_{in,BJT} = \frac{r_\pi}{g_m r_\pi + 1} \]

\[ = \frac{r_\pi}{\beta_o + 1} \]

\[ = \frac{\alpha_o}{g_m} \approx \frac{1}{g_m} \]

Input Resistance is very small!
Transistor Amplifier Configurations

Common Base Output Resistance

Replace $R_L$ by a voltage source, $v_x$

Result follows exactly after discussion in Jaeger, pages 668-670, and 683-684.
Transistor Amplifier Configurations

Common Base Output Resistance

\[ v_x = v_r + v_e \]
\[ = (i_x - \beta_o i_b) r_o + v_e \]

but,

\[ v_e = i_x \left[ r_{\pi} \parallel R_{th} \right] = i_x \frac{r_{\pi} R_{th}}{r_{\pi} + R_{th}} \]

\[ i_b = -i_x \frac{R_{th}}{r_{\pi} + R_{th}} \text{ thus,} \]

\[ v_x = i_x r_o - \beta_o \left( -i_x \frac{R_{th}}{r_{\pi} + R_{th}} \right) r_o + i_x \frac{r_{\pi} R_{th}}{r_{\pi} + R_{th}} \]

\[ \therefore R_{out,BJT} = \frac{v_x}{i_x} = r_o + \beta_o \left( \frac{R_{th}}{r_{\pi} + R_{th}} \right) r_o + \frac{r_{\pi} R_{th}}{r_{\pi} + R_{th}} \]

Output Resistance is HUGE!
Transistor Amplifier Configurations

Common Base Current Gain

\[ A_i = \frac{i_l}{i_{th}} = \alpha_o \approx 1 \]
Transistor Amplifier Configurations

Common Gate Solution

The Common Gate solution can be found by recognizing that the following translations can be made in our small signal model:

\[ \beta_o \to \infty \quad \Rightarrow \quad \alpha_o \to 1 \]

\[ r_\pi \to \infty \]

\[ A_{v,BJT} = \frac{g_m R_L}{g_m R_{th} + \left( \frac{R_{th}}{r_\pi} \right) + 1} \quad \Rightarrow \quad A_{v,MOSFET} = \frac{g_m R_L}{g_m R_{th} + 1} \]

\[ R_{in,BJT} = \frac{1}{g_m + \left( \frac{1}{r_\pi} \right)} \quad \Rightarrow \quad R_{in,MOSFET} = \frac{1}{g_m} \]

\[ R_{out,BJT} = r_o + \beta_o \left( \frac{R_{th}}{r_\pi + R_{th}} \right) r_o + \frac{r_\pi R_{th}}{r_\pi + R_{th}} = r_o + g_m r_\pi \left( \frac{R_{th}}{r_\pi + R_{th}} \right) r_o + \frac{r_\pi R_{th}}{r_\pi + R_{th}} \quad \Rightarrow \quad R_{out,MOSFET} = r_o \left( 1 + g_m R_{th} \right) + R_{th} \]

\[ A_{i,BJT} = \alpha_o \approx 1 \quad \Rightarrow \quad A_{i,BJT} = \alpha_o = 1 \]
Transistor Amplifier Configurations

Summary of Common Base and Common Gate Characteristics

- High Voltage Gain
- **Non-Inverting** Voltage Gain
- Very Low Input Impedance
- Very High Output Impedance

The input and output impedances are the opposite of what is typically needed for a voltage amplifier. Thus, Common Emitter/Source amplifiers are normally used instead of Common Base/Gate. The input and output impedances are useful for current amplifiers but the current gain is at best unity. Thus a current buffer is one useful application for the Common Base/Gate.
Multistage Amplifier Configurations

You can combine or *Cascade* configurations to produce “High Performance” amplifiers with High input impedance, low output impedance and huge voltage gains.

\[ A_v = \frac{v_o}{v_s} = \frac{v_{th,input}}{v_s} \frac{v_1}{v_1} \frac{v_2}{v_2} \frac{v_o}{v_o} \]

C-S amplifier

C-E amplifier

C-C amplifier

- **CS provides High Input Impedance, Moderately high negative gain**
- **CE provides High Input Impedance, high gain, and corrects the negative gain from previous stage**
- **CC provides Low output Impedance, no gain, but maintains positive gain from previous stage**

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For *AC-Coupled* amplifiers (capacitors between stages), the DC solution reduces to three parallel and independent circuits!
For *AC-Coupled* amplifiers (capacitors between stages), the AC solution reduces to three circuits, each of which has a load dependent on the input resistance of the next stage!

Continued…. 
Multistage Amplifier Configurations

Continued…. (For \textit{AC-Coupled} amplifiers (capacitors between stages), the AC solution reduces to three circuits, each of which has a load dependent on the input resistance of the next stage!)

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{multistage_amplifier_configurations.png}
\caption{Multistage Amplifier Configurations}
\end{figure}
Multistage Amplifier Configurations

Continued...

\[ \frac{v_{th}}{v_s} = \frac{R_G}{R_S + R_G} \quad \frac{v_1}{v_{th}} = 1 \quad \frac{v_2}{v_1} = -g_{m1}(r_{o1} \parallel R_{11} \parallel r_{\pi2}) \]

\[ \frac{v_3}{v_2} = -g_{m2}(r_{o2} \parallel R_{12} \parallel R_{mQ4}) \]

We just found this!

\[ \frac{v_3}{v_2} = -g_{m3}(r_{o2} \parallel R_{12} \parallel (r_{\pi2} + (\beta + 1)R_{L3})) \]

Note: an exact solution would have \( R_{L3} \parallel r_{o3} \)

\[ v_3 = v_o + v_4 \]

\[ v_o = v_4 \left( \frac{1}{r_{\pi3}} + g_{m3} \right) (r_{o3} \parallel R_{L3}) \]

\[ v_o = (v_3 - v_o) \left( \frac{1}{r_{\pi3}} + g_{m3} \right) (r_{o3} \parallel R_{L3}) \]

\[ v_o = \left( \frac{1}{r_{\pi3}} + g_{m3} \right) (r_{o3} \parallel R_{L3}) \]

\[ 1 + \left( \frac{1}{r_{\pi3}} + g_{m3} \right) (r_{o3} \parallel R_{L3}) \]

\[ A_v = \frac{v_o}{v_s} = \frac{v_{th} v_1 v_2 v_3 v_o}{v_s v_{th} v_1 v_2 v_3} = (1)\left( -g_{m1}(r_{o1} \parallel R_{11} \parallel r_{\pi2}) \right)\left(-g_{m2}(r_{o2} \parallel R_{12} \parallel (r_{\pi2} + (\beta + 1)R_{L3})) \right) \]

\[ \left( \frac{1}{r_{\pi3}} + g_{m3} \right) (r_{o3} \parallel R_{L3}) \]

\[ 1 + \left( \frac{1}{r_{\pi3}} + g_{m3} \right) (r_{o3} \parallel R_{L3}) \]
**AC-Coupled** amplifiers (capacitors between stages), have one major limitation. They do not amplify low frequencies or DC voltages. To accomplish this, we must **DC-Couple** the stages as shown. Note: for this to be a DC coupled amp, C1 and C6 should also be replaced as shorts.

Since the bias here is usually \((2/3)V_{cc}\) \((V_{cc} = 15V\) in this example\), it is easier to use a PNP for the second stage so that \(V_{EB} + I_E R_{E2} \approx (2/3)V_{cc}\).