Lecture 28

Operational Amplifiers

Reading: Jaeger 11.1-11.5 and Notes

Operational Amplifier

•Operational Amplifier or "Op-Amp" is a multistage amplifier that is used for general electrical signal manipulation.

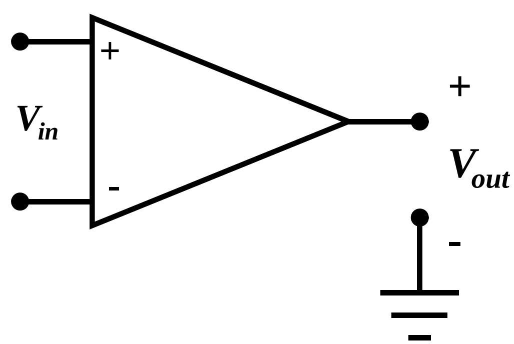
•The numbers of applications possible with Op-amps are two numerous to list.

•Most everyone agrees: "Op-Amp analysis is significantly easier than transistor analysis."

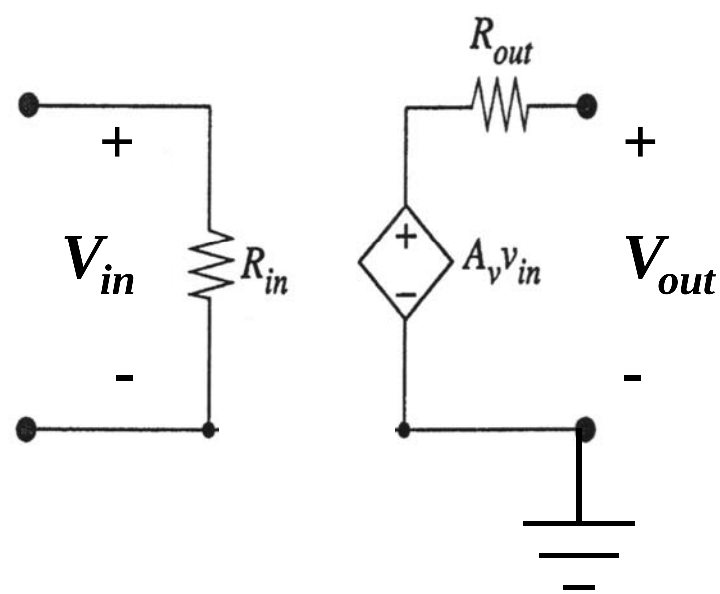
•Though they are often internally complex, their use in circuits most often simplifies the overall design.

•The circuit is modeled by an ideal voltage amplifier.

Circuit Symbol



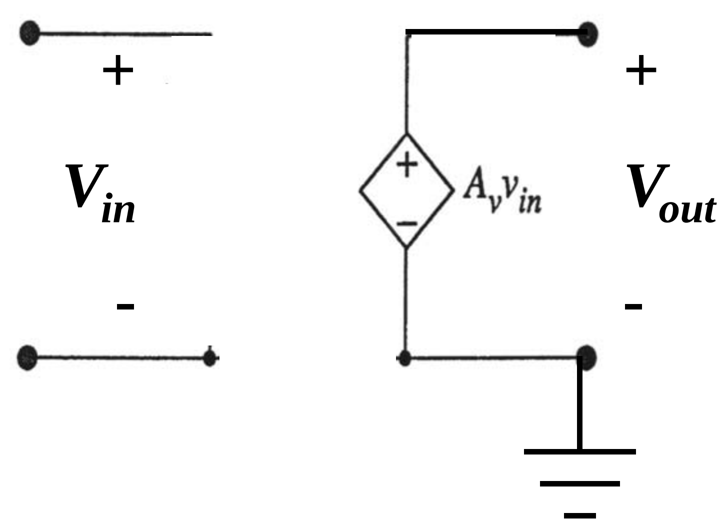
Model



Ideal Operational Amplifier

- R_{in} = Infinity,
- Voltage Gain, A_v=Infinity at all frequencies
- R_{out}=0

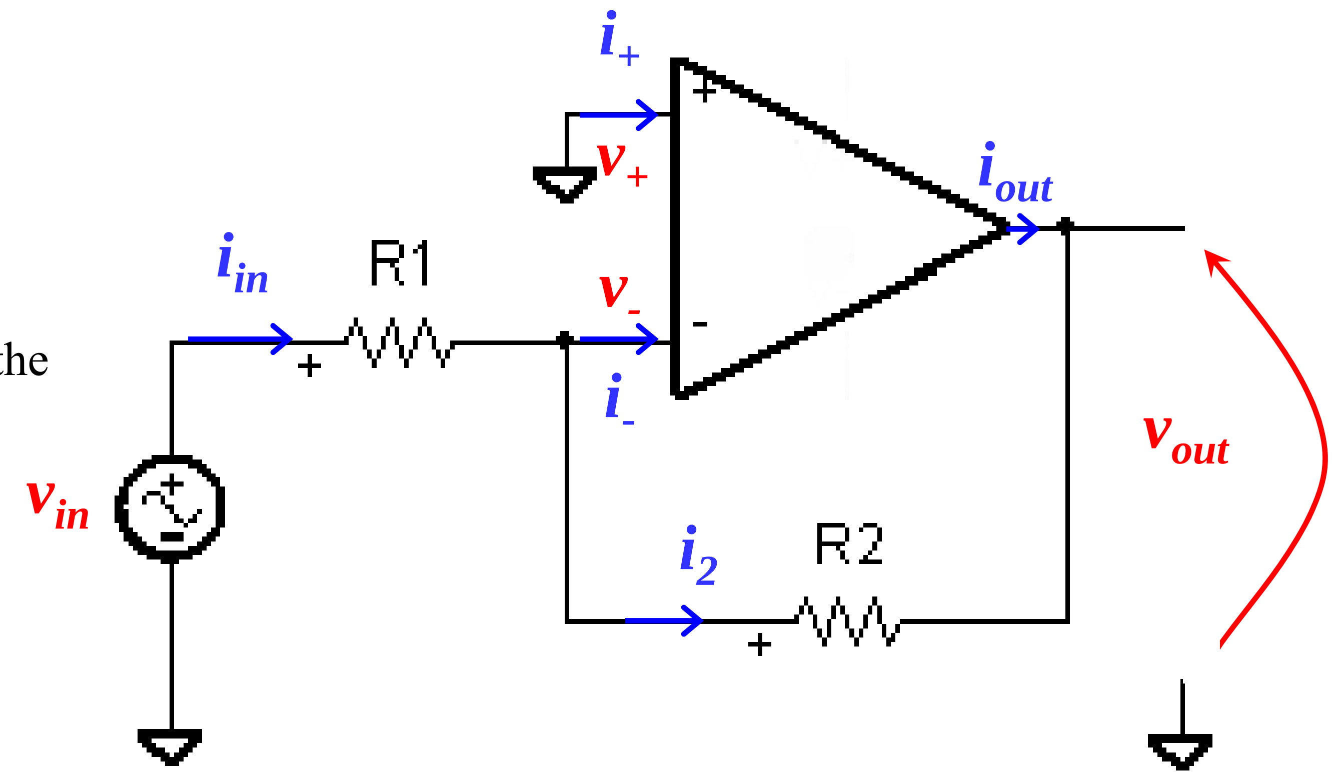
Model



Ideal Operational Amplifier

•Infinite gain means that the device is useless without adding “Feedback” to control the overall gain to a finite value.

•Consider the circuit to the right with v_{in}=0



$$v_{out} = A_v(v_+ - v_-)$$

If $A_v \rightarrow \infty$, the above equation is only satisfied for $(v_+ - v_-) = 0$

•Feedback forces the two input voltages to be equal! This is known as a “virtual ground”.

•R1 and R2 form a “Feedback Network”

Inverting Amplifier

•Finite voltage gain results from an infinite voltage gain amplifier with "negative feedback" (feedback that takes a fraction of the output voltage and mixes it back into the negative summation node).

1) $v_{in} - i_{in}R_1 - i_2R_2 - v_o = 0$

2) $i_{in} = i_- + i_2 = i_2$ due to infinite input resistance

$$\mathrm{i}_{\mathrm{in}} = \frac{\mathrm{v}_{\mathrm{in}} - \mathrm{v}_-}{\mathrm{R}_1}$$

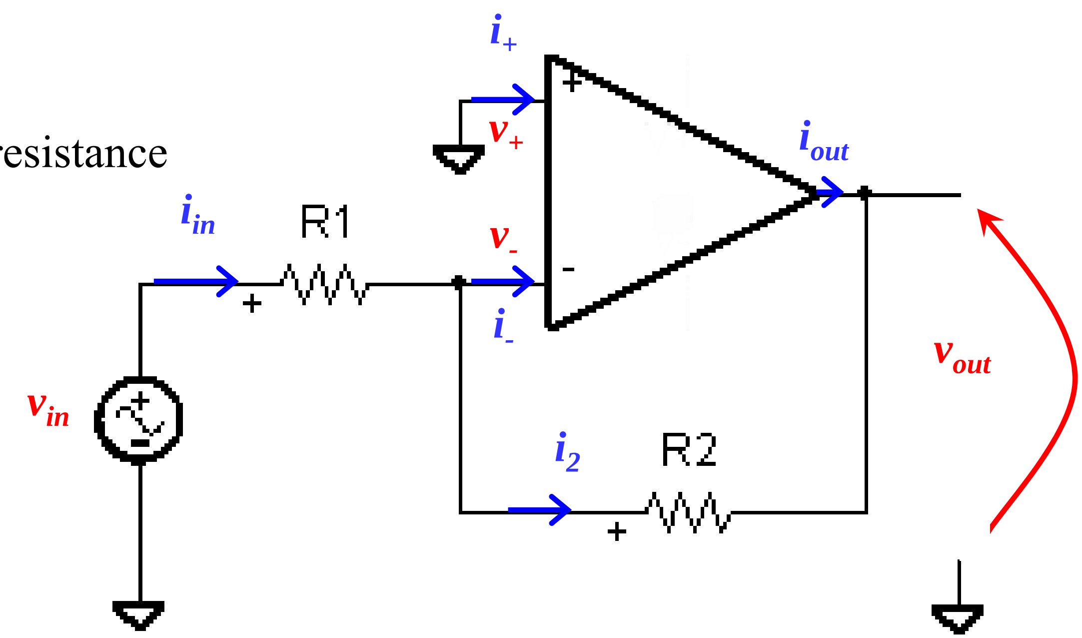
but $\mathrm{v}_- = 0$ due to the virtual ground

3) $\mathrm{i}_{\mathrm{in}} = \frac{\mathrm{v}_{\mathrm{in}}}{\mathrm{R}_1}$

Combining 1, 2 and 3,

$$v_{in} - \frac{\mathrm{v}_{\mathrm{in}}}{\mathrm{R}_1}R_1 - \frac{\mathrm{v}_{\mathrm{in}}}{\mathrm{R}_1}R_2 - v_o = 0$$

$$\boxed{\frac{v_o}{\mathrm{v}_{\mathrm{in}}} = -\frac{R_2}{R_1}}$$

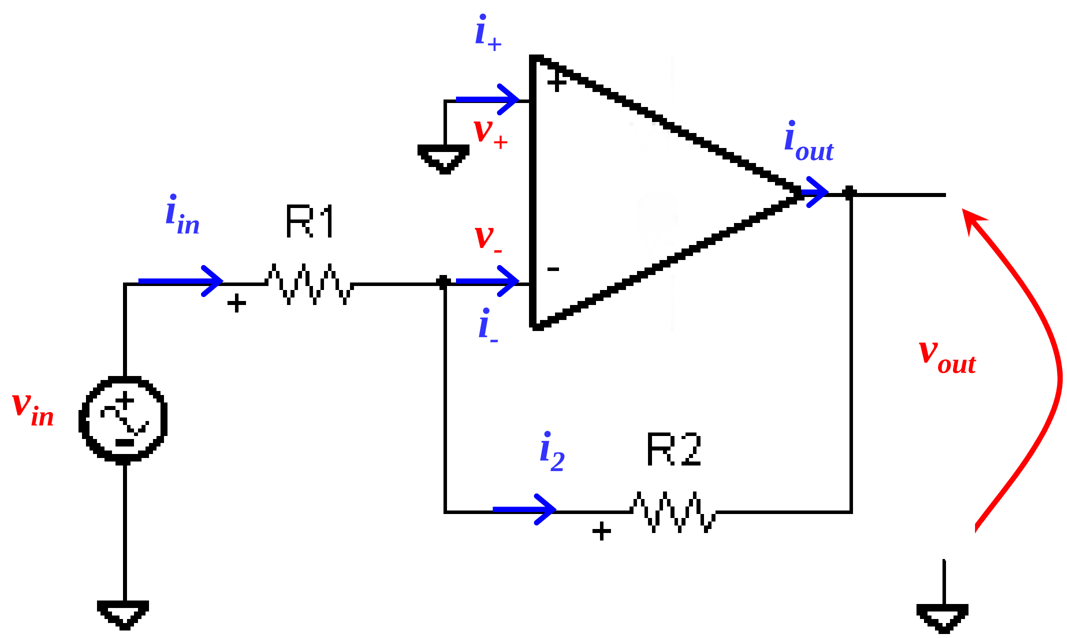


Overall circuit gain is finite, negative (for this feedback configuration) and set by the feedback resistor network.

Inverting Amplifier

•Input Resistance:

$$R_{in} = \frac{v_{in}}{i_{in}} = R_1$$



Inverting Amplifier

- Output Resistance:

$$v_t = i_1 R_3 + i_2 R_1$$

but,

$$i_1 = 0 \text{ since } v_- = 0$$

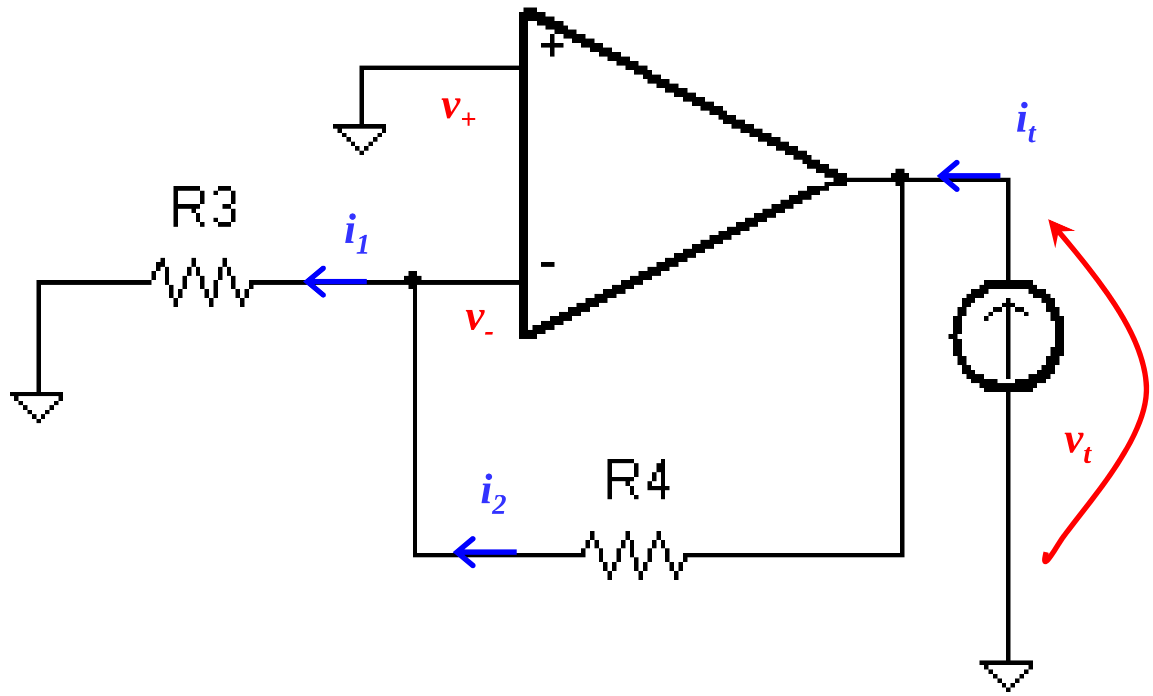
and

$$i_1 = i_2$$

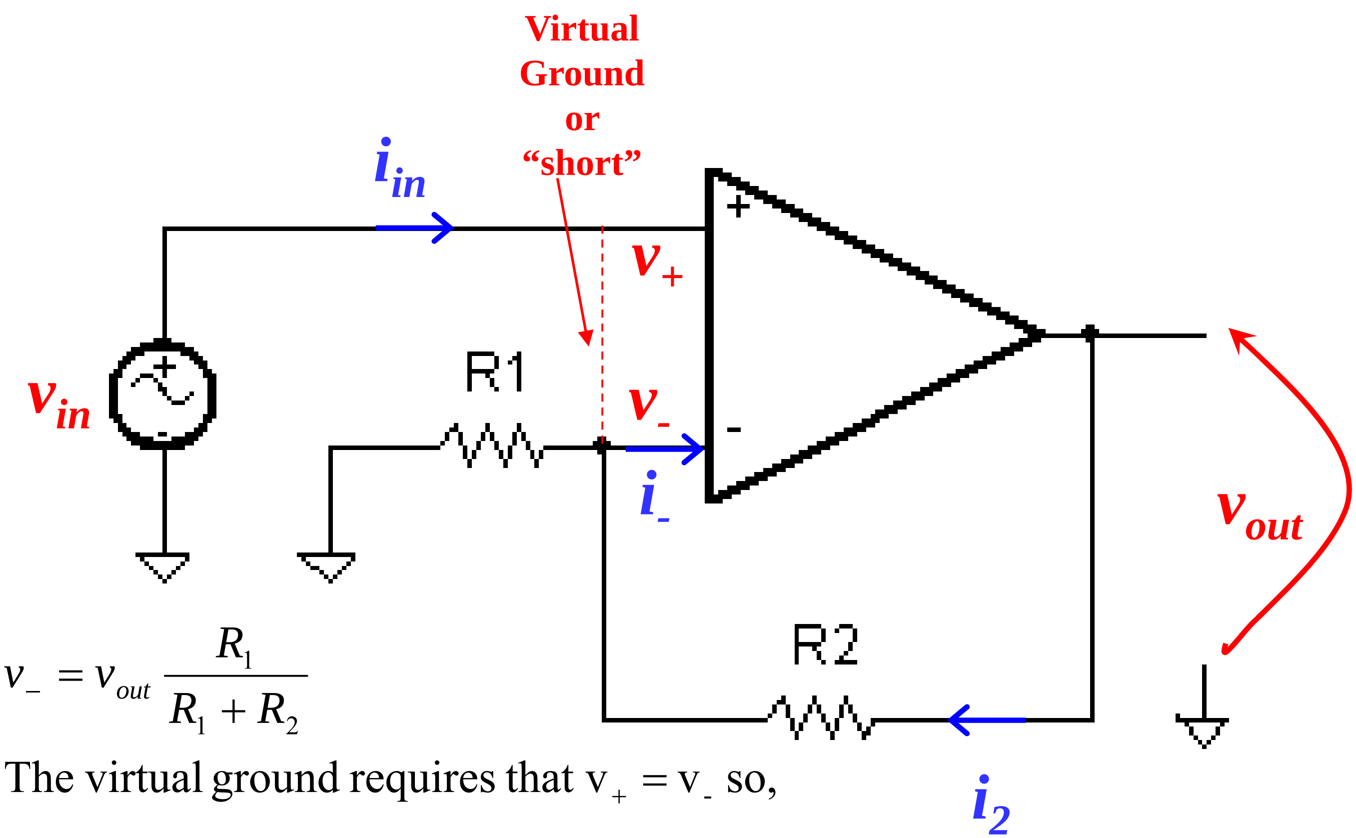
thus,

$$v_t = 0 + 0$$

$$R_{out} = \frac{v_t}{i_t} = 0$$



Non-Inverting Amplifier



$$v_- = v_{out}\frac{R_1}{R_1+R_2}$$

The virtual ground requires that $v_+ = v_-$ so,

$v_{in} = v_-$

so,

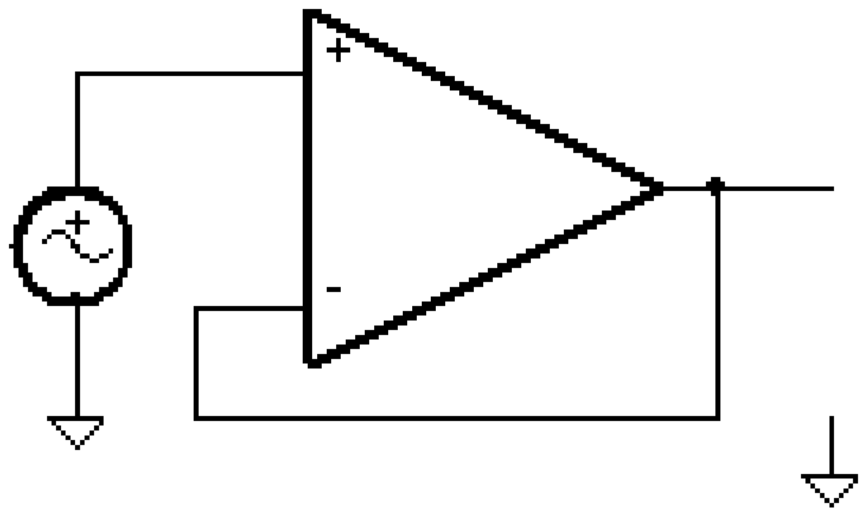
$$\frac{v_{out}}{v_{in}} = \frac{v_{out}}{v_-} = \frac{R_1+R_2}{R_1}$$

$$A_v = \frac{v_{out}}{v_{in}} = 1 + \frac{R_2}{R_1}$$

$$R_{in} = \frac{v_{in}}{i_{in}} = \frac{v_{in}}{0} = \infty$$

$R_{out} = 0$ (Same circuit as for Non-inverting case)

Unity Gain Buffer or "Voltage Follower"



$$Same\ as\ Non-inverting\ amplifier\ except\ R_2 = 0\ and\ R_1 = \infty$$

$$A_v = \frac{v_{out}}{v_-} = 1 + \frac{R_2}{R_1} \Rightarrow A_v = 1$$

$$R_{in} = \frac{v_{in}}{i_{in}} = \frac{v_{in}}{0} = \infty$$

$$R_{out} = 0 \quad \text{(Same circuit as for Non - inverting case)}$$

- Can be used to isolate a high impedance circuit from a low impedance circuit

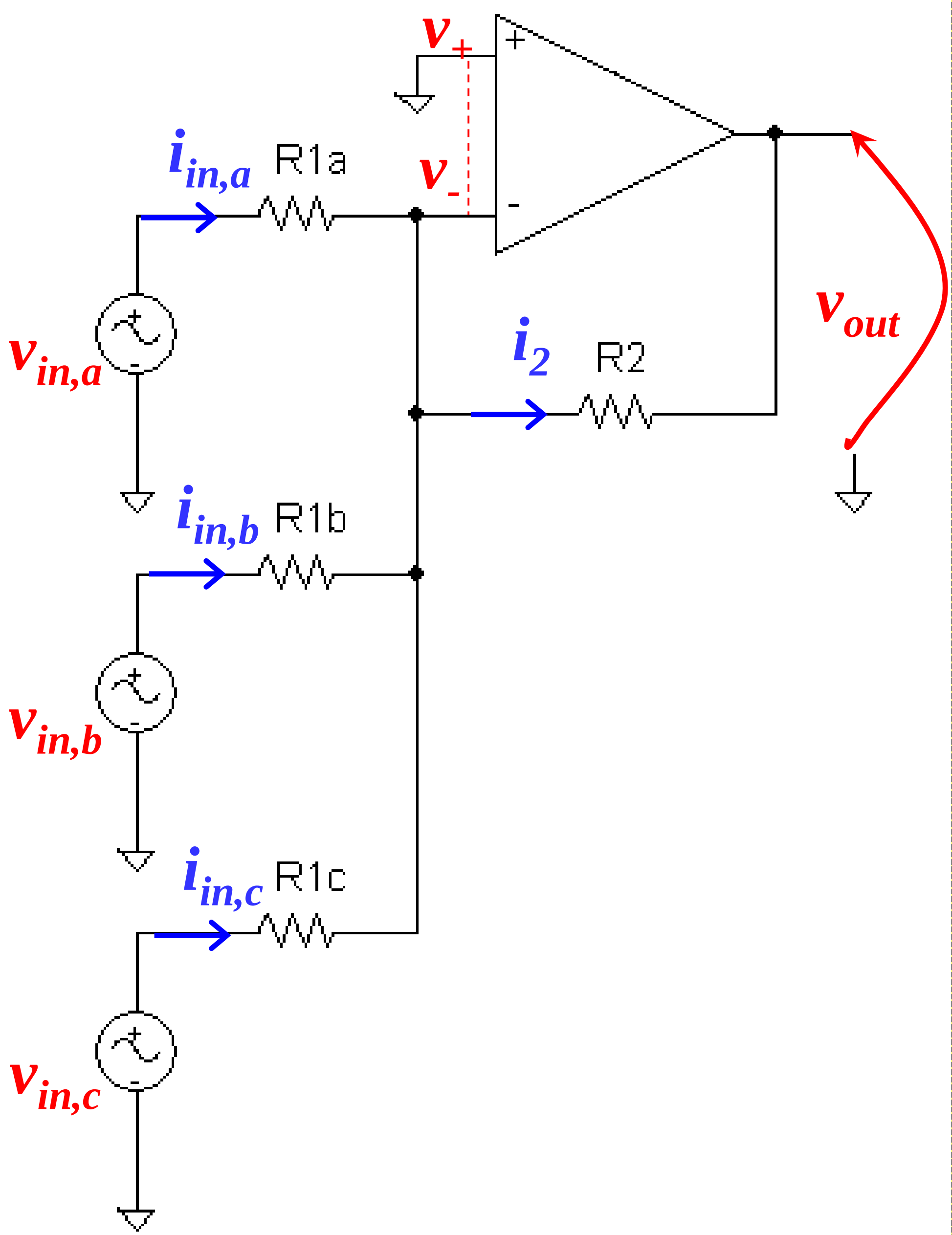
Summing Amplifier

$$i_2 = i_{in,a} + i_{in,b} + i_{in,c}$$

$$-\frac{v_{out}}{R2} = \frac{v_{in,a}}{R1a} + \frac{v_{in,b}}{R1b} + \frac{v_{in,c}}{R1c}$$

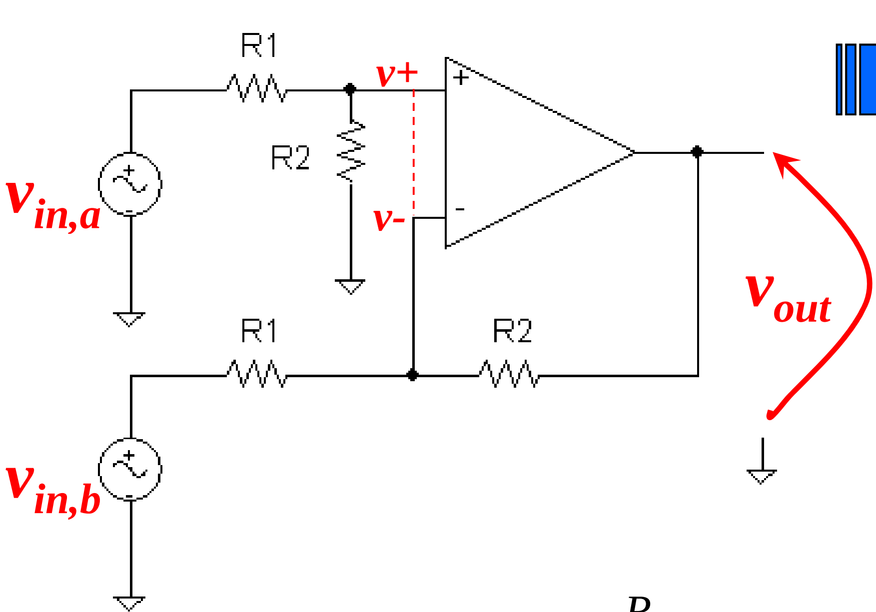
$$v_{out} = -v_{in,a}\frac{R2}{R1a} - v_{in,b}\frac{R2}{R1b} - v_{in,c}\frac{R2}{R1c}$$

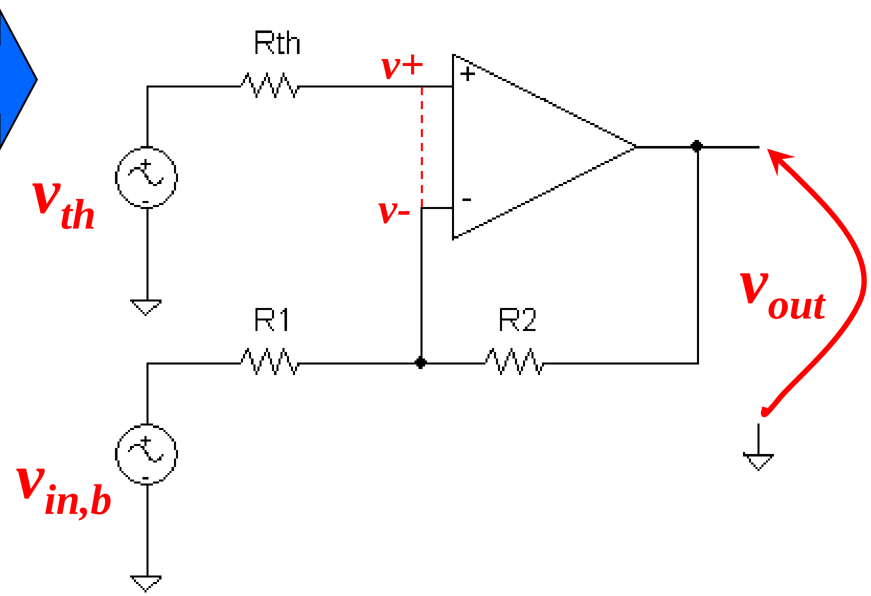
$$v_{out} = -\left(v_{in,a}\frac{R2}{R1a} + v_{in,b}\frac{R2}{R1b} + v_{in,c}\frac{R2}{R1c}\right)$$



- Output is a scaled sum of inputs.
- Scaling can be controlled by ratios of resistors

Difference Amplifier





$$v_{th} = v_{in,a}\frac{R_2}{R_1 + R_2} \qquad and \qquad R_{th} = R_1 \parallel R_2$$

Using Superposition we can combine the results of the Inverting and Non-inverting solutions:

$v_{in,b}$=0

$$v_{out} = v_{th}\left(1 + \frac{R_2}{R_1}\right)$$

$$v_{out} = v_{in,a}\frac{R_2}{R_1 + R_2}\left(1 + \frac{R_2}{R_1}\right)$$

$$v_{out} = v_{in,a}\frac{R_2}{R_1}$$

$v_{in,a}$=0

$$v_{out} = -\mathrm{v}_{\mathrm{in,b}}\frac{R_2}{R_1}$$

$$v_{out} = \mathrm{v}_{\mathrm{in,a}}\frac{R_2}{R_1} - \mathrm{v}_{\mathrm{in,b}}\frac{R_2}{R_1}$$

$$v_{out} = \left(\mathrm{v}_{\mathrm{in,a}} - \mathrm{v}_{\mathrm{in,b}}\right)\frac{R_2}{R_1}$$

This circuit amplifies the difference of two signals

Non-Ideal (Real World) Operational Amplifiers

Finite Open-Loop Gain

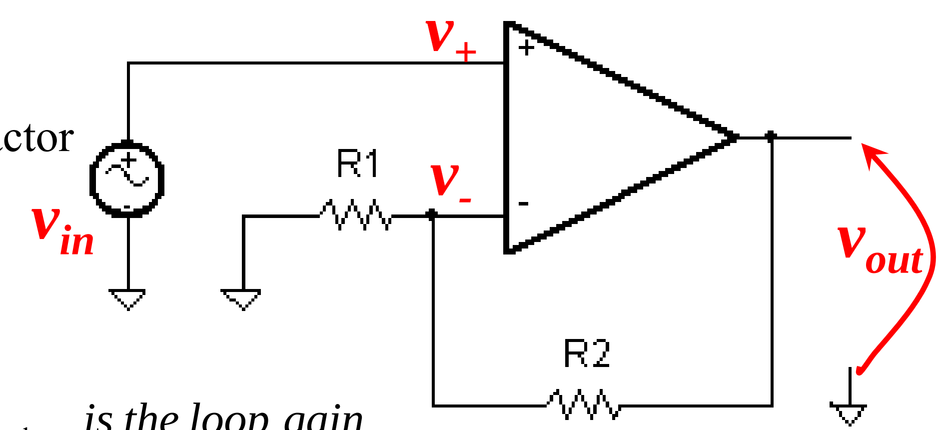
•Real op-amps do not have "infinite" "open loop (without feedback)" gain.

•Voltage gains are typically large but finite: ~10^4-10^6 V/V

•Finite gain causes a deviation from ideal amplifier behavior

$$v_- = v_{out}\frac{R_1}{R_1+R_2} = \beta v_{out}$$

where $\beta = \dfrac{R_1}{R_1+R_2}$ is known as the feedback factor



$$v_{out} = A_{openloop}\left(v_+ - v_-\right) = A_{openloop}\left(v_+ - \beta v_{out}\right)$$

so since $v_+ = v_{in}$,

$$A_{v,\,closed\,loop} = \frac{v_{out}}{v_{in}} = \frac{A_{openloop}}{1+\beta A_{openloop}},\ \textit{where}\ \beta A_{openloop}\ \textit{is the loop gain}$$

$$\textit{If}\quad \beta A_{openloop} >> 1$$

$$A_{v,\,closed\,loop} = \frac{1}{\beta} = 1 + \frac{R_2}{R_1} \quad \Rightarrow \quad \text{approaches the infinite gain result}$$

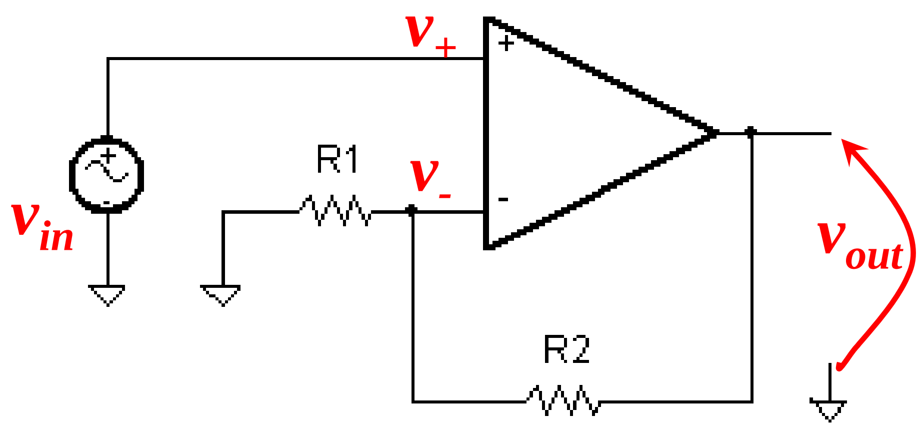
Non-Ideal (Real World) Operational Amplifiers

Finite Open-Loop Gain

•Finite open-loop gain means the Virtual Ground is not perfect!

$$v_+ - v_- = v_{in} - \beta v_{out} = v_{in} - \beta\left(\frac{\mathrm{A}_{\mathrm{openloop}}}{1+\beta\,\mathrm{A}_{\mathrm{openloop}}}\right)v_{in} = \frac{v_{in}}{1+\beta\,\mathrm{A}_{\mathrm{openloop}}}$$

Small but finite offset between + and - terminals



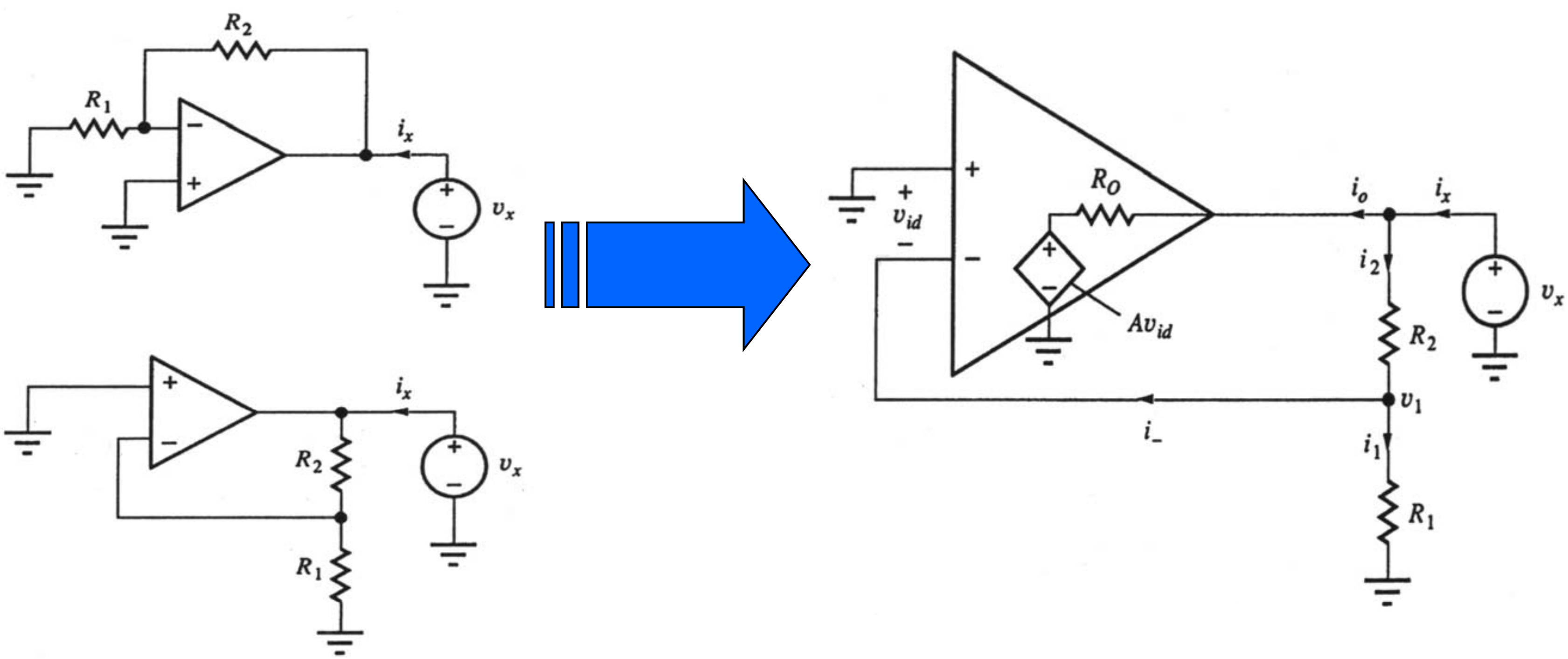
•The Gain Error (GE) that results from the Non-infinite open-loop gain can be quantified as:

$$GE = \left(\frac{1}{\beta}\right) - \left(\frac{\mathrm{A}_{\mathrm{openloop}}}{1+\beta\,\mathrm{A}_{\mathrm{openloop}}}\right)v_{in} = \frac{v_{in}}{\beta\left(1+\beta\,\mathrm{A}_{\mathrm{openloop}}\right)}$$

Non-Ideal (Real World) Operational Amplifiers

Finite Output Impedance

- Real Op-Amps have a small but finite output impedance, Ro .
- We want to find the Output impedance of the various circuits we have examined .
- All the configurations have a common circuit for calculating the output impedance .



Non-Ideal (Real World) Operational Amplifiers

Finite Output Impedance

$$R_{out} = \frac{v_x}{i_x}$$

$$i_x = i_o + i_2$$

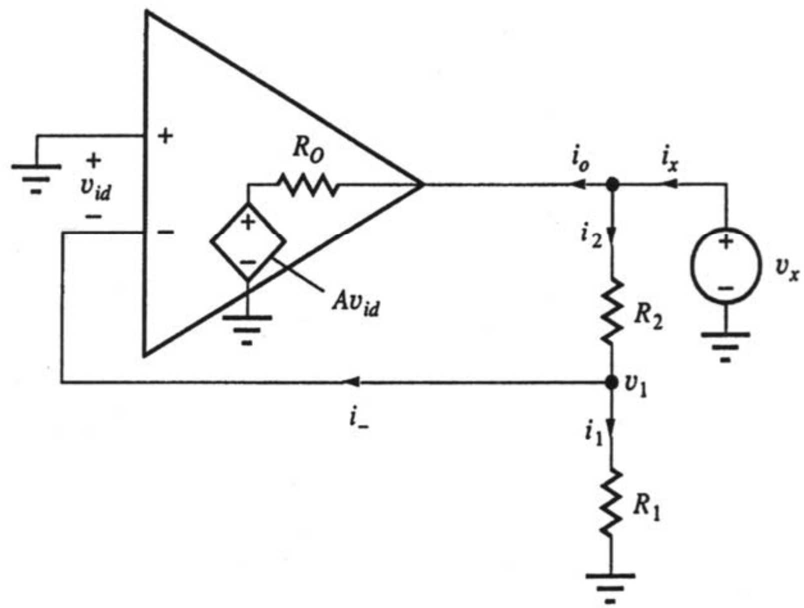
$$i_o = \frac{v_x - A_{v,openloop}\left(v_+ - v_-\right)}{R_o}$$

$$v_x = i_2\left(R_1 + R_2\right) \quad \Rightarrow \quad i_2 = \frac{v_x}{\left(R_1 + R_2\right)}$$

$$v_- = \frac{R_1}{\left(R_1 + R_2\right)} v_x = \beta v_x$$

$$\left(R_{out}\right)^{-1} = \frac{i_x}{v_x} = \frac{1 + A_{v,openloop}\beta}{R_o} + \frac{1}{\left(R_1 + R_2\right)}$$

$$R_{out} = \underbrace{\left(\frac{R_o}{1 + A_{v,openloop}\beta}\right)} \parallel \left(R_1 + R_2\right)$$



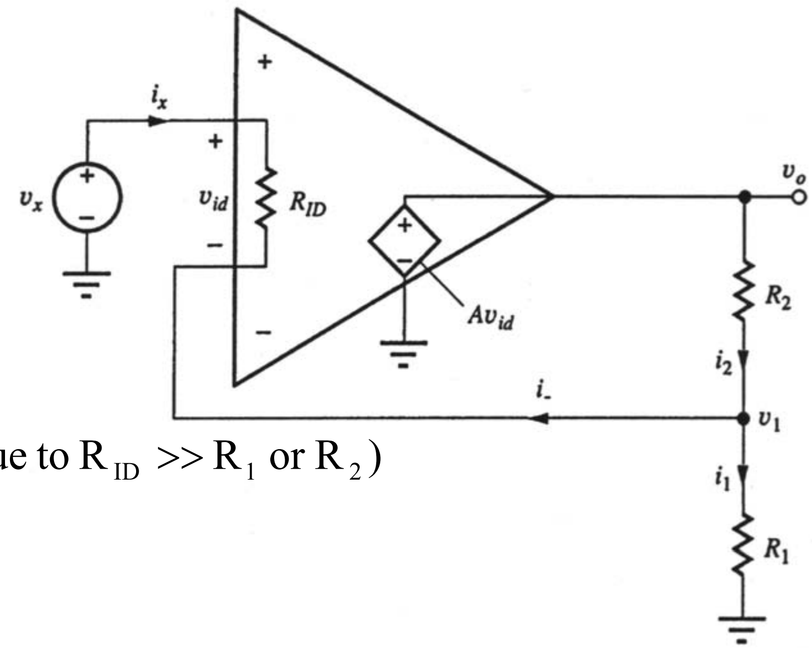
R_o is very small so this term is *EXTREMELY* small!

FEEDBACK REDUCES OUTPUT IMPEDANCE!

Non-Ideal (Real World) Operational Amplifiers

Finite Input Impedance: Non-Inverting Case

•Real Op-Amps have a large but finite input resistance, R_{ID}



$$i_x = \frac{v_x - v_-}{R_{ID}}$$

Neglecting the current i_x compared to i_1 and i_2 (due to $R_{ID} >> R_1$ or R_2)

$$v_- = i_1 R_1 \approx i_2 R_1$$

$$v_- = \frac{R_1}{R_1 + R_2} v_{out} = \beta v_{out} = A_{v,openloop}\beta (v_x - v_-)$$

$$v_- = \frac{A_{v,openloop}\beta}{1 + A_{v,openloop}\beta} v_x$$

$$i_x = \frac{v_x - \left(\frac{A_{v,openloop}\beta}{1 + A_{v,openloop}\beta} v_x\right)}{R_{ID}} = \frac{v_x}{(1 + A_{v,openloop}\beta)R_{ID}}$$

$$R_{in} = \underbrace{(1 + A_{v,openloop}\beta)R_{ID}}$$

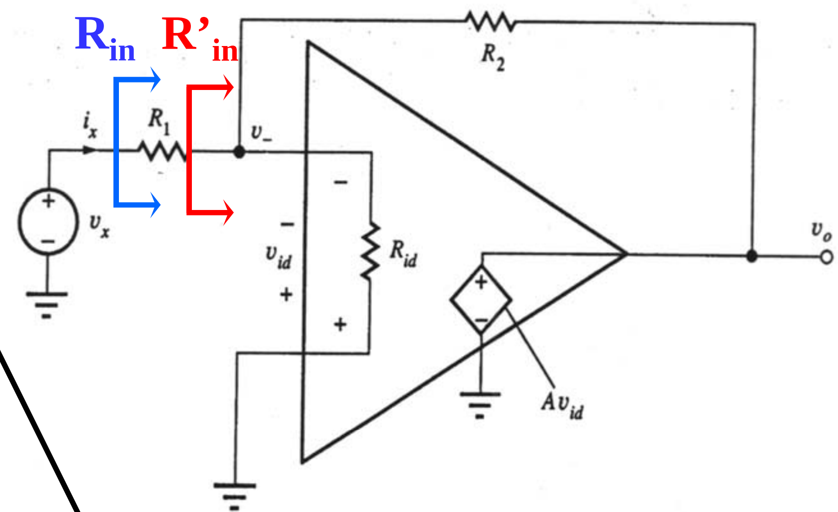
FEEDBACK INCREASES INPUT IMPEDANCE!

R_{ID} is very large so R_{in} is *EXTREMELY* large!

Non-Ideal (Real World) Operational Amplifiers

Finite Input Impedance: Inverting Case

- Real Op-Amps have a large but finite input resistance, R_{ID}
- $\mathbf{R_{in}}=R_1+\mathbf{R'_{in}}$
- Find R'_{in} by forming a new test circuit



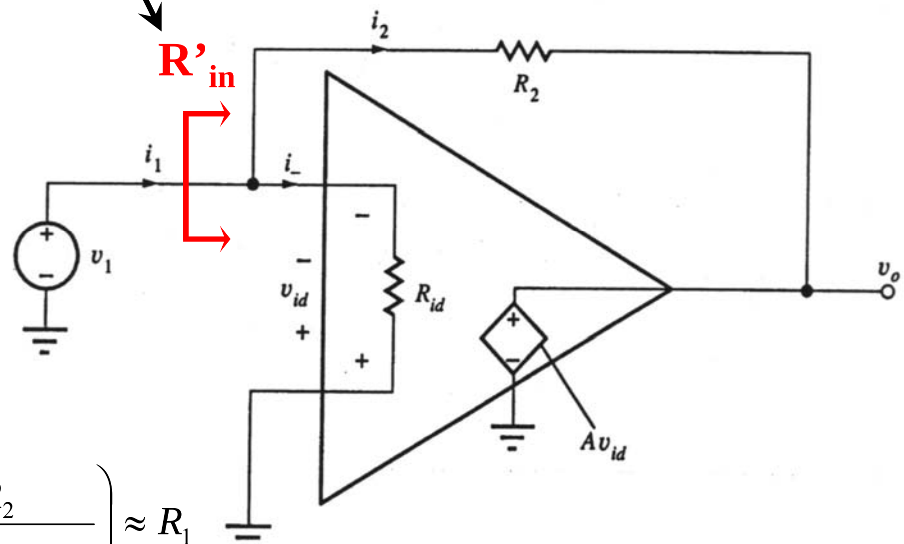
$$i_1 = i_- + i_2 = \frac{v_1}{R_{ID}} + \frac{v_1 - v_{out}}{R_2} = \frac{v_1}{R_{ID}} + \frac{v_1 + A_{v,openloop} v_1}{R_2}$$

$$(R'_{in})^{-1} = \frac{i_1}{v_1} = \frac{1}{R_{ID}} + \frac{1 + A_{v,openloop}}{R_2}$$

$$R'_{in} = (R_{ID}) \Bigg\| \left(\frac{R_2}{1 + A_{v,openloop}} \right)$$

Thus,

$$R_{in} = R_1 + (R_{ID}) \Bigg\| \left(\frac{R_2}{1 + A_{v,openloop}} \right)$$



Since $R_{ID} >> R_2/(1+A_{v,openloop})$ and $A_{v,openloop}$ is very large,

$$R_{in} = R_1 + (R_{ID}) \Bigg\| \left(\frac{R_2}{1 + A_{v,openloop}} \right) \Rightarrow R_{in} = R_1 + \left(\frac{R_2}{1 + A_{v,openloop}} \right) \approx R_1$$