Lecture 29

Operational Amplifier frequency Response

Reading: Jaeger 12.1 and Notes
Ideal Op Amps Used to Control Frequency Response

Low Pass Filter

Previously:

\[
\frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1}
\]

Now put a capacitor in parallel with R2:

If \( s = j\omega \),

\[
\frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1} \left[ \frac{1}{C_2 s} \right]
\]

\[
\frac{V_{out}}{V_{in}} = -\frac{1}{R_1} \left[ \frac{R_2}{C_2 s} \right] = -\frac{R_2}{R_1} \left( \frac{1}{1 + R_2 C_2 s} \right)
\]
Ideal Op Amps Used to Control Frequency Response

Low Pass Filter

\[ \frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1} \left( \frac{1}{1 + \frac{R_2C_2s}{R_1}} \right) \]

- At DC (s=0), the gain remains the same as before (-R_2/R_1).

- At high frequency, R_2C_2s >> 1, the gain dies off with increasing frequency,

  \[ \frac{V_{out}}{V_{in}} \approx -\left( \frac{1}{R_1C_2s} \right) = -\left( \frac{1}{\frac{C_2s}{R_1}} \right) \]

  \[ \frac{1}{R_2C_2} = 2\pi f_H = \omega_H \]

- At high frequencies, more “negative feedback” reduces the overall gain.
Ideal Op Amps Used to Control Frequency Response

Low Pass Filter

\[
|A_V|_{DB} = 20 \log \left( \frac{R_2}{R_1} \right)
\]

is the gain expressed in dB

-3dB drop at \(f_H\) (\(V_{out}\) has dropped in half!)

\[f_H = \frac{1}{2\pi R_2 C_2}\]

- At DC (\(s=0\)), the gain remains the same as before (-\(R_2/R_1\))
- At high frequency, \(R_2C_2s >> 1\), the gain dies off with increasing frequency
- Implements a “Low Pass Filter”: Lower frequencies are allowed to pass the filter without attenuation. High frequencies are strongly attenuated (do not pass).
Ideal Op Amps Used to Control Frequency Response

High Pass Filter

\[
\frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1}
\]

- At DC (s=0), the gain is zero.
- At high frequency, \( R_1C_1s \gg 1 \), the gain returns to its full value, \((-R_2/R_1)\)
Ideal Op Amps Used to Control Frequency Response

High Pass Filter

- At DC \((s=0)\), the gain is zero.
- At high frequency, \(R_1C_1s >> 1\), the gain returns to it’s full value, \((-\frac{R_2}{R_1})\)
- Implements a “High Pass Filter”: Higher frequencies are allowed to pass the filter without attenuation. Low frequencies are strongly attenuated (do not pass).

\[
\left| A_V \right|_{DB} = 20 \log \left( \frac{R_2}{R_1} \right)
\]

\[
\frac{V_{out}}{V_{in}} = \frac{R_2C_1s}{1 + R_1C_1s}
\]

\[
f_L = \frac{1}{2\pi R_1C_1}
\]
Ideal Op Amps Used to Control Frequency Response

Band Pass Filter (combination of high and low pass filter)

\[
\frac{V_{out}}{V_{in}} = -\frac{R_2 \left( \frac{1}{C_2 s} \right)}{R_1 + \frac{1}{C_1 s}}
\]

Low Pass

\[
\frac{V_{out}}{V_{in}} = -\frac{R_2 + \frac{1}{C_2 s}}{R_1 + \frac{1}{C_1 s}} = -\left( \frac{1}{1 + R_2 C_2 s} \right) \left( \frac{R_2 C_1 s}{1 + R_1 C_1 s} \right)
\]

High Pass
Ideal Op Amps Used to Control Frequency Response

Band Pass Filter (combination of high and low pass filter)

\[
\frac{V_{out}}{V_{in}} = -\left(1 + R_2 C_2 s\right) \left(1 + R_1 C_1 s\right)
\]

Slopes = $\pm 20 \text{ dB/Decade}$

-3dB drop at $f_H$

\[
|A_V|_{DB} = 20 \log\left(\frac{R_2}{R_1}\right)
\]

\[
f_L = \frac{1}{2\pi R_1 C_1}
\]

\[
f_H = \frac{1}{2\pi R_2 C_2}
\]

$f_L << f_H$
Ideal Op Amps Used to Control Frequency Response

Band Pass Filter (combination of high and low pass filter)

\[
\frac{V_{out}}{V_{in}} = -\left(\frac{1}{1 + R_2 C_2 s}\right)\left(\frac{R_2 C_1 s}{1 + R_1 C_1 s}\right)
\]

Slopes = ±20 dB / Decade

\[|A_v|_{DB} = 20 \log \left(\frac{R_2}{R_1}\right)\]

More than a -3dB drop at \(f_L\) and \(f_H\)

\[f_L < f_H \quad \text{and} \quad f_L \rightarrow f_H\]
General Frequency Response of a Circuit

Poles and Zeros

Generally, a circuit’s transfer function (frequency dependent gain expression) can be written as the ratio of polynomials:

\[
\frac{v_{out}}{v_{in}} = A \frac{(\tau_{1z} s)(1 + \tau_{2z} s)(1 + \tau_{3z} s) \cdots}{(1 + \tau_{1p} s)(1 + \tau_{2p} s)(1 + \tau_{3p} s) \cdots}
\]

Complex Roots of the numerator polynomial are called “zeros” while complex roots of the denominator polynomial are called “poles”

Each zero causes the transfer function to “break to higher gain” (slope increases by 20 dB/decade)

Each pole causes the transfer function to “break to lower gain” (slope decreases by 20 dB/decade)

Typically, \( \tau = RC \)
Real Op Amp Frequency Response

- To this point we have assumed the open loop gain, $A_{\text{Open Loop}}$, of the op amp is constant at all frequencies.

- Real Op amps have a frequency dependant open loop gain.

$$A_{\text{Open Loop}}(s) = \frac{A_O \omega_B}{s + \omega_B} = \frac{\omega_T}{s + \omega_B}$$

where,

$s = j \omega$

$A_O \equiv$ Open loop gain at DC

$\omega_B \equiv$ Open loop bandwidth

$\omega_T \equiv$ Unity - gain frequency \(\left(\text{frequency where } |A_{\text{Open Loop}}(s)| = 1\right)\)
Real Op Amp Frequency Response

At Low Frequencies: \[ |A_{\text{OpenLoop}}| = A_O \]

At High Frequencies: \[ |A_{\text{OpenLoop}}| \approx \frac{A_O \bar{\omega} B}{\bar{\omega}} = \frac{\bar{\omega}_T}{\bar{\omega}} \]

For most frequencies of interest, \( \omega \gg \omega_B \), the product of the gain and frequency is a constant, \( \omega_T \)

\[ f_T = \frac{\bar{\omega}_T}{2\pi} \equiv \text{Gain – Bandwidth Product (GBW)} \]
Real Op Amp Frequency Response

For the "741" Op Amp,

\[ A_O \sim 200,000 = 106 \, dB \]
\[ \omega_B \sim (2\pi) \, 5 \, Hz \]
\[ \omega_T \sim (2\pi) \, 1 \, MHz \]

For the "Op 07" Op Amp,

\[ A_O \sim 12,000,000 = 141 \, dB \]
\[ \omega_B \sim (2\pi) \, 0.05 \, Hz \]
\[ \omega_T \sim (2\pi) \, 0.6 \, MHz \]

If the open loop bandwidth is so small, how can the op amp be useful?

The answer to this is found by considering the closed loop gain.
Real Op Amp Frequency Response

Previously, we found that the closed loop gain for the Non-inverting configuration was (for finite open loop gain):

\[
A_{V,\text{ClosedLoop}} = \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{A_{\text{OpenLoop}}}{1 + \beta A_{\text{OpenLoop}}}, \text{ where } \beta = \frac{R_1}{R_1 + R_2}
\]

Using the frequency dependent open loop gain:

\[
A_{V,\text{ClosedLoop}} = \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{A_{\text{OpenLoop}}}{1 + \beta A_{\text{OpenLoop}}}
\]

\[
A_{V,\text{ClosedLoop}} = \frac{\left(\frac{A_O \sigma_B}{s + \sigma_B}\right)}{\left(1 + \frac{A_O \sigma_B}{s + \sigma_B}\right)} = \frac{A_O \sigma_B}{s + \sigma_B (1 + \beta A_O)}
\]

\[
A_{V,\text{ClosedLoop}} = \frac{\sigma_B (1 + \beta A_O)}{s} + 1 = \frac{A_O}{s (1 + \beta A_O)} + 1 = \left(\frac{1}{1 + \frac{s}{\sigma_H}}\right) A_{V,\text{ClosedLoop}} |_{DC}
\]

where,

\[\sigma_H \equiv \text{Upper Cutoff Frequency (Closed Loop Bandwidth)} = \sigma_B (1 + \beta A_O)\]
The closed Loop Amplifier has a lower gain than the Open Loop Amplifier.

Closed Loop Bandwidth

$$\omega_H = \omega_B (1 + \beta A_O) = \frac{\omega_T}{A_{V,\text{ClosedLoop}}|_{\text{@ DC}}}$$

Closed Loop DC Gain

$$A_{V,\text{ClosedLoop}} = \frac{A_{\text{OpenLoop}}}{1 + \beta A_{\text{OpenLoop}}}$$
Real Op Amp Frequency Response

\[
\left( \text{Gain} \times \text{Bandwidth} \right)_{\text{Open Loop}} = \left( \text{Gain} \times \text{Bandwidth} \right)_{\text{Closed Loop}}
\]

Example: 741 Op Amp is used as a low pass filter with \( f_L = 10\,\text{kHz} \). What is the maximum voltage gain possible for this circuit?

From before, we can write:

\[
(200,000 \times 5)_{\text{Open Loop}} = (\text{Gain} \times 10,000)_{\text{Closed Loop}}
\]

\[
(\text{Gain})_{\text{Closed Loop}} = 100 \frac{V}{V} \text{ Maximum}
\]
Real Op Amp Frequency Response

For the Inverting Configuration:

By superposition,

\[ v_\text{in} = v_\text{out} \frac{R_1}{R_1 + R_2} + v_\text{in} \frac{R_2}{R_1 + R_2} \]

\[ v_\text{in} = v_\text{out} \beta + v_\text{in} \beta \frac{R_2}{R_1} \]

but,

\[ v_\text{out} = -v_\text{in} A_{V,\text{OpenLoop}} \]

so,

\[ -\frac{v_\text{out}}{A_{V,\text{OpenLoop}}} = v_\text{out} \beta + v_\text{in} \beta \frac{R_2}{R_1} \]

\[ A_{V,\text{ClosedLoop}} = \frac{v_\text{out}}{v_\text{in}} = \frac{A_{V,\text{OpenLoop}} \beta}{1 + A_{V,\text{OpenLoop}} \beta} \left( -\frac{R_2}{R_1} \right) \]
Real Op Amp Frequency Response

Inserting the frequency dependent open loop gain:

\[ A_{V,\text{ClosedLoop}} = \frac{A_{V,\text{OpenLoop}} \beta}{1 + A_{V,\text{OpenLoop}} \beta} \left( -\frac{R_2}{R_1} \right) \]

\[ A_{V,\text{ClosedLoop}} = \left( \frac{A_O \omega_B}{s + \omega_B} \right) \beta \left( -\frac{R_2}{R_1} \right) = \frac{A_O \omega_B \beta}{s + \omega_B + A_O \omega_B \beta} \left( -\frac{R_2}{R_1} \right) \]

\[ A_{V,\text{ClosedLoop}} = \frac{A_O \omega_B \beta}{s + \omega_B (1 + A_O \beta)} \left( -\frac{R_2}{R_1} \right) = \frac{A_O \omega_B \beta}{\omega_B (1 + A_O \beta)} \left( -\frac{R_2}{R_1} \right) \]

\[ A_{V,\text{ClosedLoop}} = \left( \frac{A_O \beta}{(1 + A_O \beta)} \left( -\frac{R_2}{R_1} \right) \right) \]

\[ = \frac{s}{\omega_B (1 + A_O \beta) + 1} \]
Real Op Amp Frequency Response

\[
A_{V,\text{ClosedLoop}} = \left( \frac{A_O \beta}{(1 + A_O \beta)} \left( \frac{-R_2}{R_1} \right) \right) \left( \frac{s}{\omega_B (1 + A_O \beta)} + 1 \right)
\]

\[
A_{V,\text{ClosedLoop}} = \left( \frac{1}{1 + \frac{s}{\omega_B (1 + A_O \beta)}} \right) A_{V,\text{ClosedLoop}} \bigg|_{@\, DC}
\]

Closed Loop Bandwidth

\[
\omega_H = \omega_B (1 + \beta A_O) = \omega_T \bigg|_{A_{V,\text{ClosedLoop}} \, @\, DC}
\]

Closed Loop DC Gain

\[
A_{V,\text{ClosedLoop}} = \frac{A_{V,\text{OpenLoop}} \beta}{1 + A_{V,\text{OpenLoop}} \beta} \left( -\frac{R_2}{R_1} \right)
\]

The frequency behavior is the same as for the the Non-Inverting case!