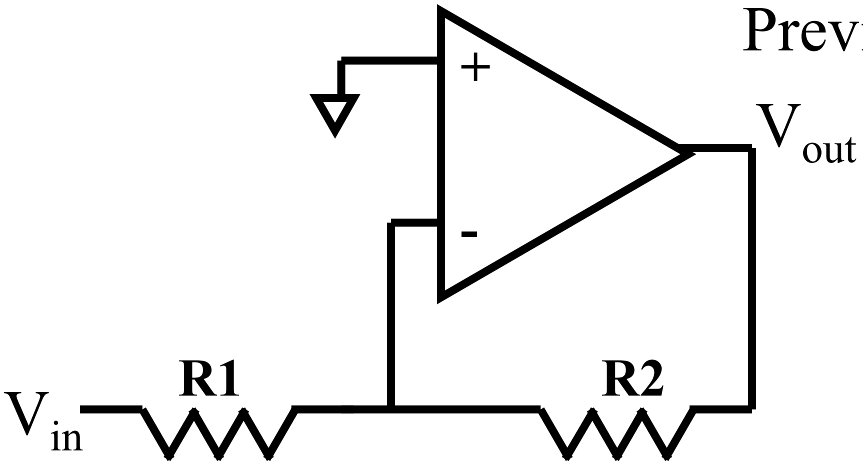
Lecture 29

Operational Amplifier frequency Response

Reading: Jaeger 12.1 and Notes

Ideal Op Amps Used to Control Frequency Response

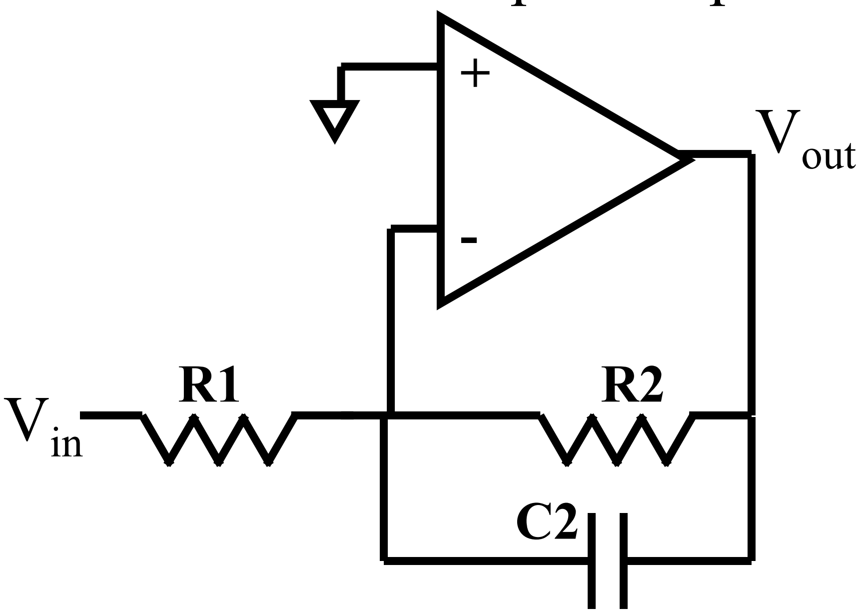
Low Pass Filter



Previously:

$$\frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1}$$

Now put a capacitor in parallel with R2:



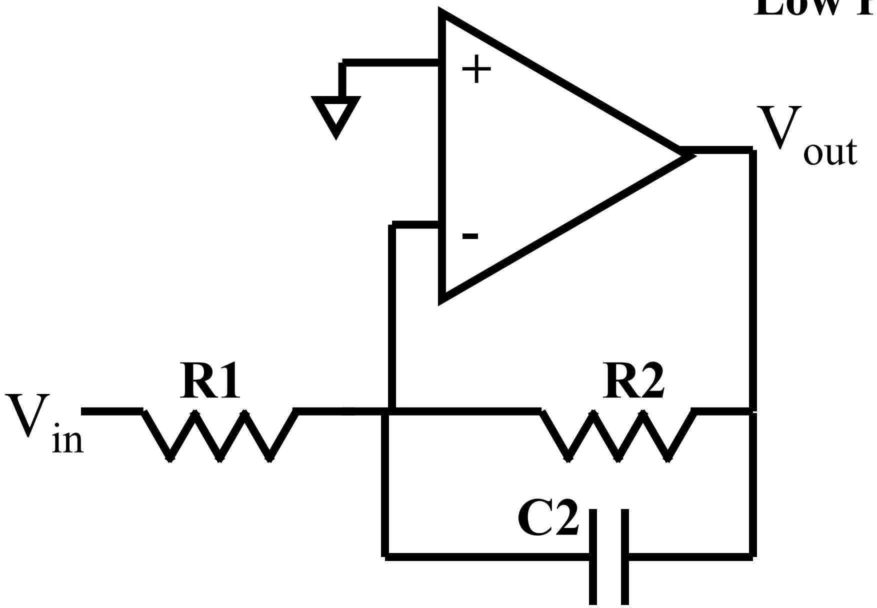
$$If \;\; s = j\varpi,$$

$$\frac{V_{out}}{V_{in}} = -\frac{R_2 \left\| \frac{1}{C_2 s} \right.}{R_1}$$

$$\frac{V_{out}}{V_{in}} = -\frac{1}{R_1}\frac{R_2 \frac{1}{C_2 s}}{R_2 + \frac{1}{C_2 s}} = -\frac{R_2}{R_1}\left(\frac{1}{1 + R_2 C_2 s}\right)$$

Ideal Op Amps Used to Control Frequency Response

Low Pass Filter



$$\frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1}\left(\frac{1}{1+R_2C_2s}\right)$$

- At DC (s=0), the gain remains the same as before ($-R_2/R_1$).
- At high frequency, $R_2C_2s>>1$, the gain dies off with increasing frequency,

$$\frac{V_{out}}{V_{in}} \approx -\left(\frac{1}{R_1C_2s}\right) = -\left(\frac{\frac{1}{C_2s}}{R_1}\right)$$

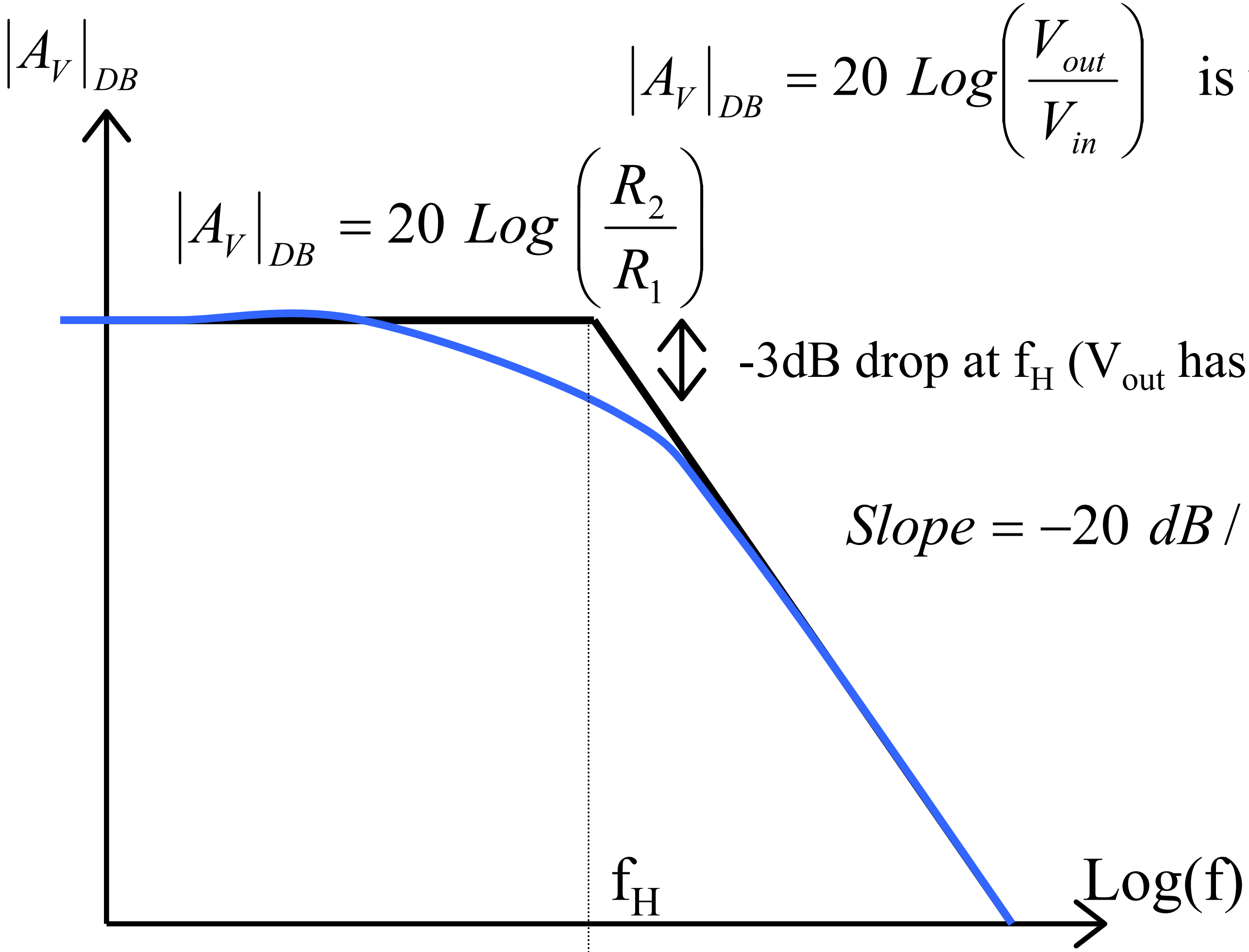
$$\frac{1}{R_2C_2} = 2\pi f_H = \omega_H$$

- At high frequencies, more "negative feedback" reduces the overall gain

Ideal Op Amps Used to Control Frequency Response

Low Pass Filter

$$\left|A_V\right|_{DB} = 20\ Log\left(\frac{V_{out}}{V_{in}}\right)$$ is the gain expressed in dB



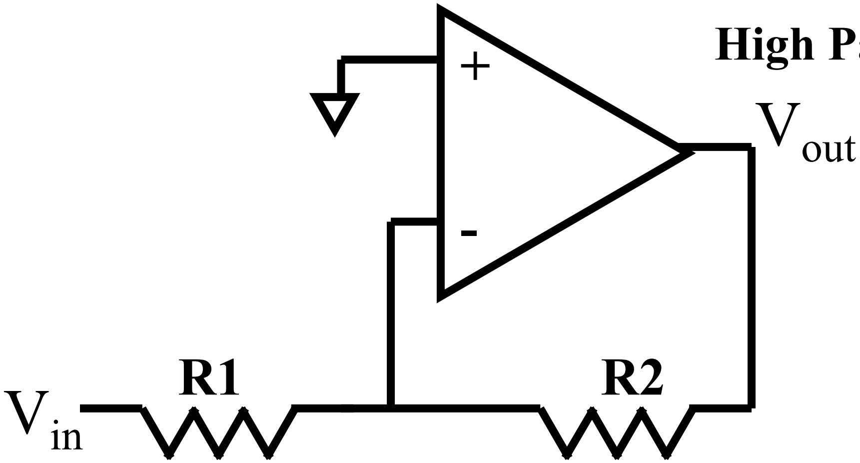
$$f_H = \frac{1}{2\pi\ R_2 C_2}$$

- At DC (s=0), the gain remains the same as before (-R2/R1)
- At high frequency, $R_2C_2s>>1$, the gain dies off with increasing frequency

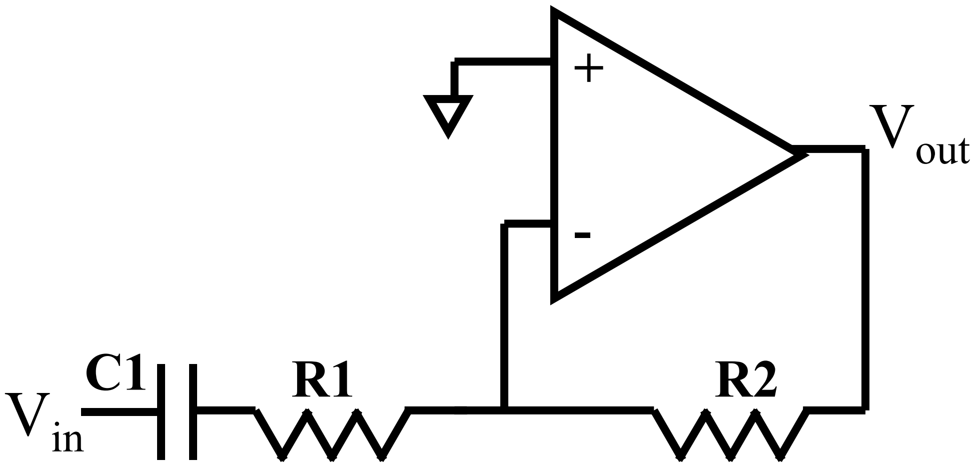
- Implements a "Low Pass Filter": Lower frequencies are allowed to pass the filter without attenuation. High frequencies are strongly attenuated (do not pass).

Ideal Op Amps Used to Control Frequency Response

High Pass Filter



$$\frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1}$$



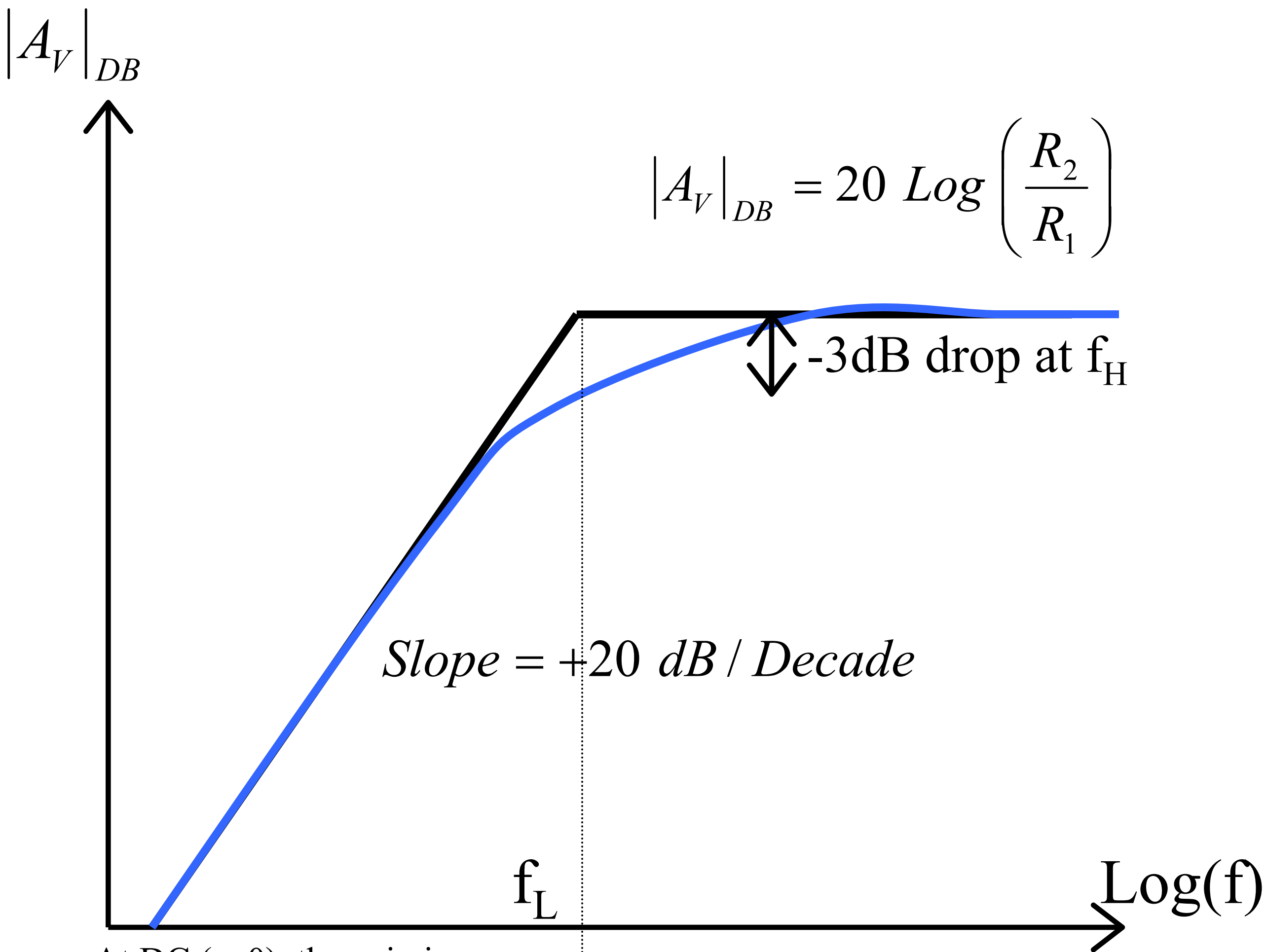
$$\frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1 + \frac{1}{C_1 s}}$$

$$\frac{V_{out}}{V_{in}} = -\frac{R_2 C_1 s}{1 + R_1 C_1 s}$$

- At DC (s=0), the gain is zero.
- At high frequency, $R_1C_1s>>1$, the gain returns to it's full value, $(-R_2/R_1)$

Ideal Op Amps Used to Control Frequency Response

High Pass Filter



$$\frac{V_{out}}{V_{in}} = -\frac{R_2 C_1 s}{1 + R_1 C_1 s}$$

$$f_L = \frac{1}{2\pi\ R_1 C_1}$$

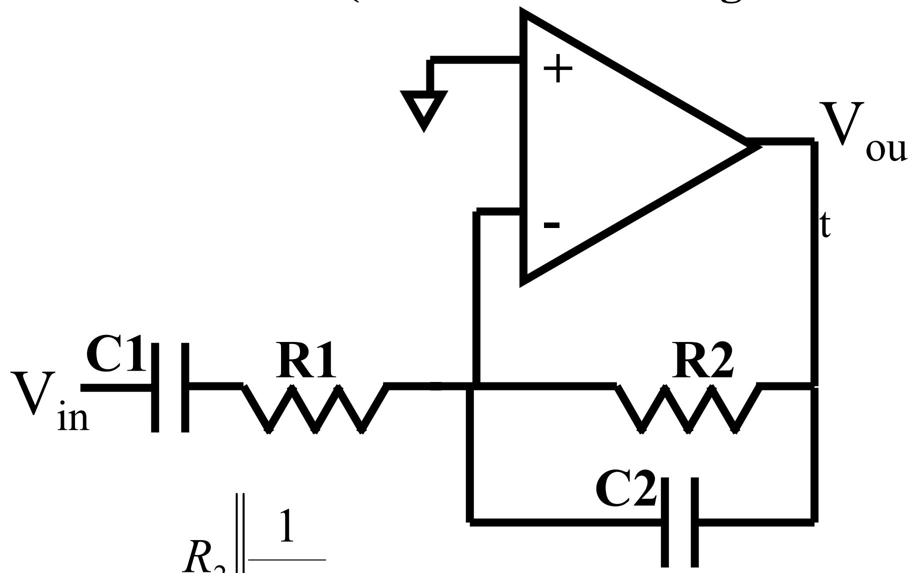
•At DC (s=0), the gain is zero.

•At high frequency, $R_1C_1s>>1$, the gain returns to it's full value, $(-R_2/R_1)$

•Implements a "High Pass Filter": Higher frequencies are allowed to pass the filter without attenuation. Low frequencies are strongly attenuated (do not pass).

Ideal Op Amps Used to Control Frequency Response

Band Pass Filter (combination of high and low pass filter)



$$\frac{V_{out}}{V_{in}} = -\frac{R_2 \left\| \frac{1}{C_2 s} \right.}{R_1 + \frac{1}{C_1 s}}$$

$$\frac{V_{out}}{V_{in}} = -\frac{\dfrac{R_2 \frac{1}{C_2 s}}{R_2 + \frac{1}{C_2 s}}}{R_1 + \frac{1}{C_1 s}} = -\overbrace{\left(\frac{1}{1 + R_2 C_2 s}\right)}^{\text{Low Pass}}\overbrace{\left(\frac{R_2 C_1 s}{1 + R_1 C_1 s}\right)}^{\text{High Pass}}$$

Ideal Op Amps Used to Control Frequency Response

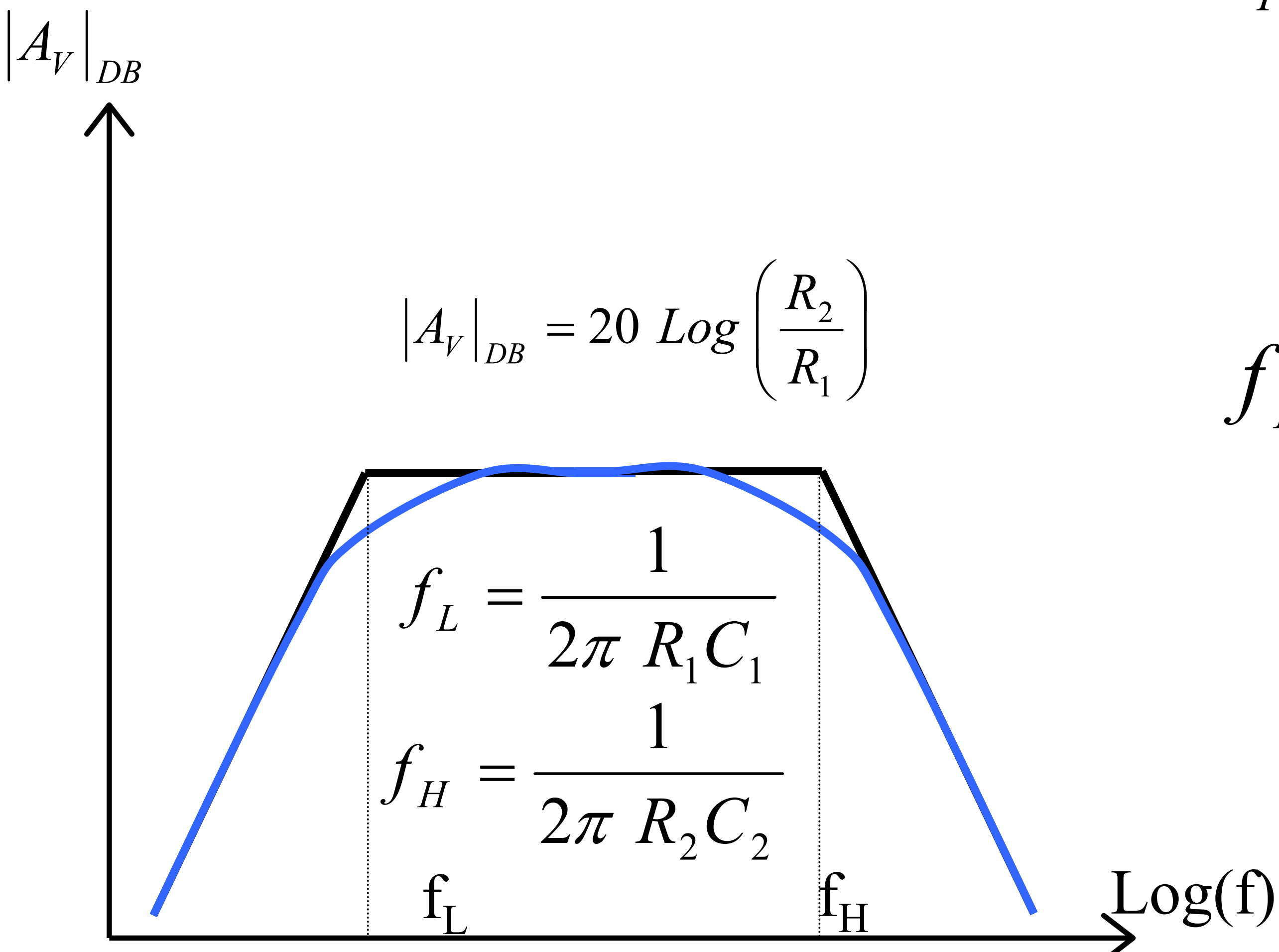
Band Pass Filter (combination of high and low pass filter)

$$\frac{V_{out}}{V_{in}} = -\left(\frac{1}{1+R_2C_2s}\right)\left(\frac{R_2C_1s}{1+R_1C_1s}\right)$$

$$Slopes = \mp 20\ dB / Decade$$

↕ -3dB drop at f_H

$$f_L << f_H$$



$$\left|A_V\right|_{DB} = 20\ Log\left(\frac{R_2}{R_1}\right)$$

$$f_L = \frac{1}{2\pi\ R_1C_1}$$

$$f_H = \frac{1}{2\pi\ R_2C_2}$$

Ideal Op Amps Used to Control Frequency Response

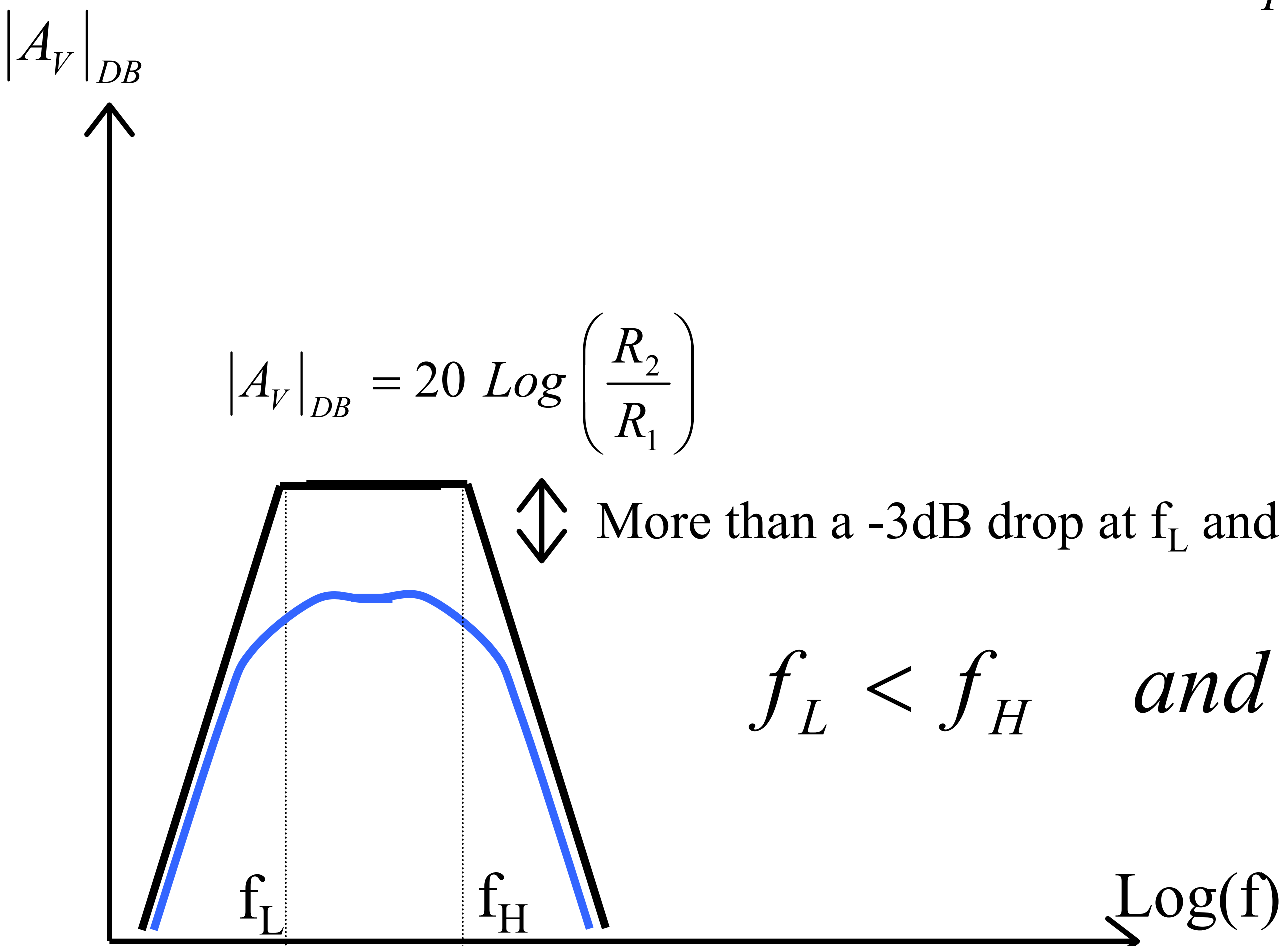
Band Pass Filter (combination of high and low pass filter)

$$\frac{V_{out}}{V_{in}} = -\left(\frac{1}{1+R_2C_2s}\right)\left(\frac{R_2C_1s}{1+R_1C_1s}\right)$$

$$Slopes = {}^{-}\!/_{+}\, 20\ dB/Decade$$

$$\left|A_V\right|_{DB} = 20\ Log\left(\frac{R_2}{R_1}\right)$$

More than a -3dB drop at f_L and f_H



$$f_L < f_H \quad and \quad f_L \rightarrow f_H$$

General Frequency Response of a Circuit

Poles and Zeros

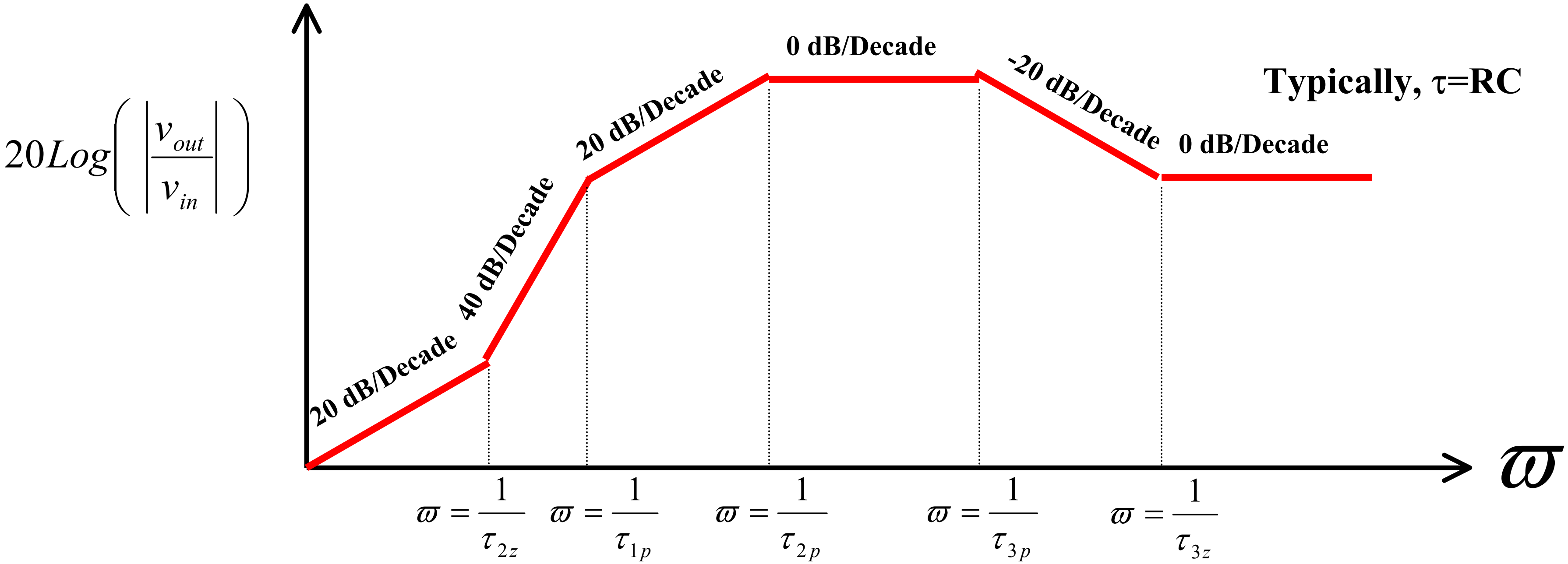
Generally, a circuit's transfer function (frequency dependent gain expression) can be written as the ratio of polynomials:

$$\frac{v_{out}}{v_{in}} = A\frac{(\tau_{1z}s)(1+\tau_{2z}s)(1+\tau_{2z}s)\ldots}{(1+\tau_{1p}s)(1+\tau_{2p}s)(1+\tau_{3p}s)\ldots} \Rightarrow \left|\frac{v_{out}}{v_{in}}\right| = A\frac{(\tau_{1z}\varpi)\sqrt{1-(\tau_{2z}\varpi)^2}\sqrt{1-(\tau_{3z}\varpi)^2}\ldots}{\sqrt{1-(\tau_{1p}\varpi)^2}\sqrt{1-(\tau_{2p}\varpi)^2}\sqrt{1-(\tau_{3p}\varpi)^2}\ldots}$$

Complex Roots of the numerator polynomial are called "zeros" while complex roots of the denominator polynomial are called "poles"

Each zero causes the transfer function to "break to higher gain" (slope increases by 20 dB/decade)

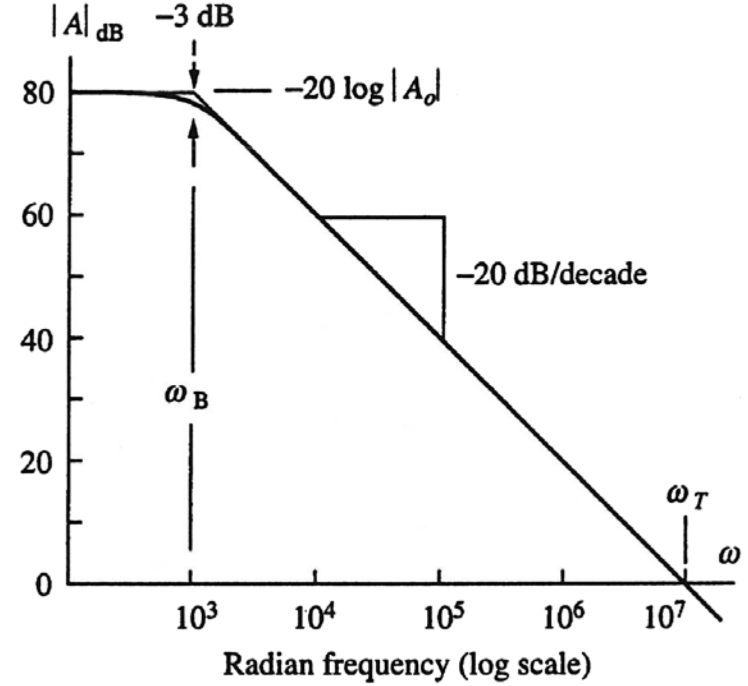
Each pole causes the transfer function to "break to lower gain" (slope decreases by 20 dB/decade)



Real Op Amp Frequency Response

•To this point we have assumed the open loop gain, $A_{\text{Open Loop}}$, of the op amp is constant at all frequencies.

•Real Op amps have a frequency dependant open loop gain.



$$A_{OpenLoop}(s) = \frac{A_O \varpi_B}{s + \varpi_B} = \frac{\varpi_T}{s + \varpi_B}$$

where,

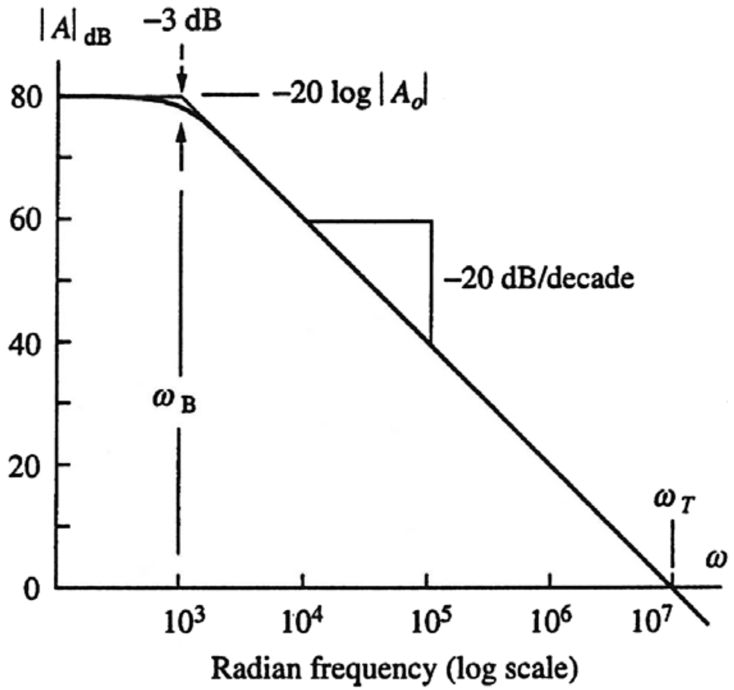
$$s = j\varpi$$

$$A_O \equiv \text{Open loop gain at DC}$$

$$\varpi_B \equiv \text{Open loop bandwidth}$$

$$\varpi_T \equiv \text{Unity - gain frequency}\left(\text{frequency where}\left|A_{OpenLoop}(s)\right| = 1\right)$$

Real Op Amp Frequency Response



$$\left|A_{OpenLoop}(j\varpi)\right| = \frac{A_O \varpi_B}{\sqrt{\varpi^2 + \varpi_B^{\ 2}}}$$

$$\left|A_{OpenLoop}(j\varpi)\right| = \frac{A_O}{\sqrt{1 + \dfrac{\varpi^2}{\varpi_B^{\ 2}}}}$$

At Low Frequencies: $\left|A_{OpenLoop}\right| = A_O$

At High Frequencies: $\left|A_{OpenLoop}\right| \approx \dfrac{A_O \varpi_B}{\varpi} = \dfrac{\varpi_T}{\varpi}$

For most frequencies of interest, $\omega >> \omega_B$, the product of the gain and frequency is a constant, ω_T

$$f_T = \frac{\varpi_T}{2\pi} \equiv Gain - Bandwidth\ \mathrm{Pr}oduct\ (GBW)$$

Real Op Amp Frequency Response

For the "741" Op Amp,

$A_O \sim 200{,}000 = 106\ dB$

$\varpi_B \sim (2\pi)\,5\ \text{Hz}$

$\varpi_T \sim (2\pi)\underbrace{1\ \text{MHz}}_{\textbf{GBW}}$

For the "Op 07" Op Amp,

$A_O \sim 12{,}000{,}000 = 141\ dB$

$\varpi_B \sim (2\pi)\,0.05\ \text{Hz}$

$\varpi_T \sim (2\pi)\underbrace{0.6\ \text{MHz}}_{\textbf{GBW}}$

If the open loop bandwidth is so small, how can the op amp be useful?

The answer to this is found by considering the closed loop gain.

Real Op Amp Frequency Response

Previously, we found that the closed loop gain for the Non-inverting configuration was (for finite open loop gain):

$$A_{V,ClosedLoop} = \frac{V_{out}}{V_{in}} = \frac{A_{OpenLoop}}{1+\beta\ A_{OpenLoop}}, \text{ where } \beta = \frac{R_1}{R_1 + R_2}$$

Using the frequency dependent open loop gain:

$$A_{V,ClosedLoop} = \frac{V_{out}}{V_{in}} = \frac{A_{OpenLoop}}{1+\beta\ A_{OpenLoop}}$$

$$A_{V,ClosedLoop} = \frac{\left(\dfrac{A_O\varpi_B}{s+\varpi_B}\right)}{1+\beta\left(\dfrac{A_O\varpi_B}{s+\varpi_B}\right)} = \frac{A_O\varpi_B}{s+\varpi_B\left(1+\beta\ A_O\right)}$$

Low Pass

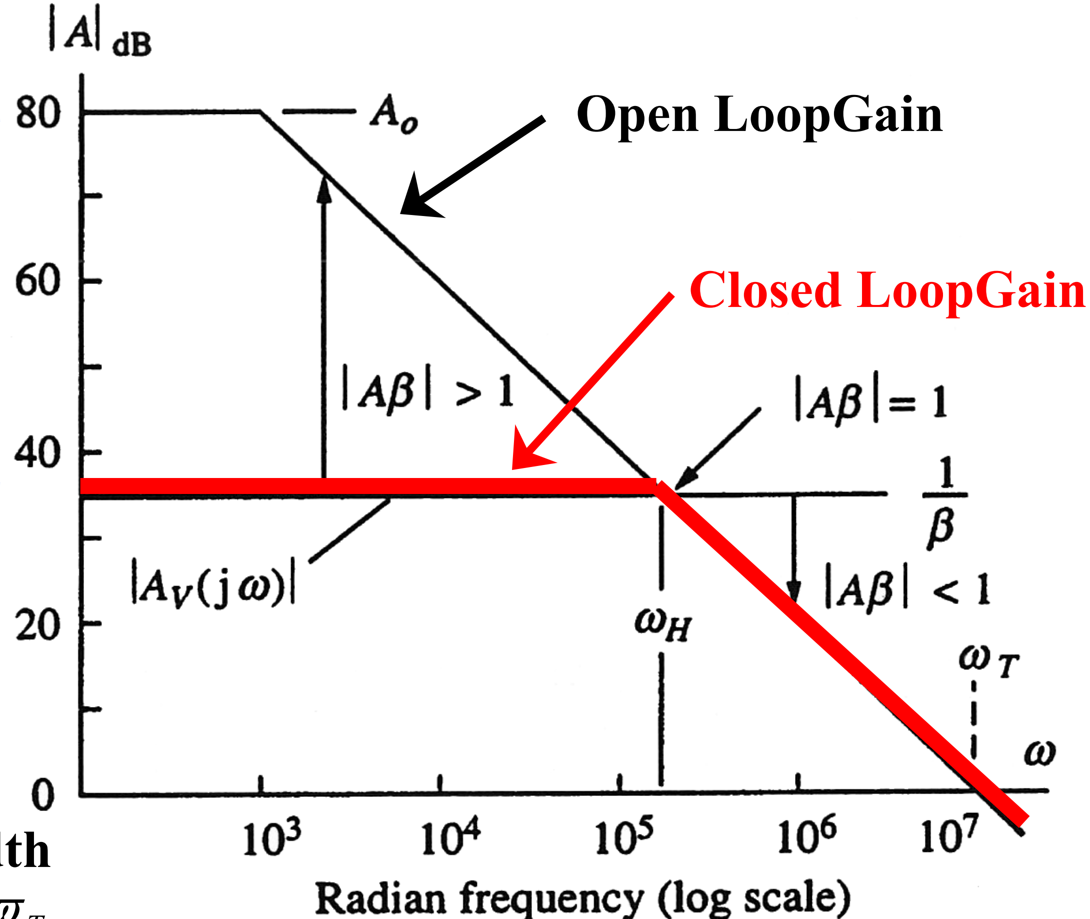
$$A_{V,ClosedLoop} = \frac{\dfrac{A_O\varpi_B}{\varpi_B\left(1+\beta\ A_O\right)}}{\dfrac{s}{\varpi_B\left(1+\beta\ A_O\right)}+1} = \frac{\dfrac{A_O}{\left(1+\beta\ A_O\right)}}{\dfrac{s}{\varpi_B\left(1+\beta\ A_O\right)}+1} = \overbrace{\left(\frac{1}{1+\dfrac{s}{\varpi_H}}\right)}^{\text{Low Pass}} A_{V,ClosedLoop}\Big|_{@\,DC}$$

where,

$$\varpi_H \equiv Upper\ Cutoff\ \ Frequency\ \left(Closed\ \ Loop\ \ Bandwith\right) = \varpi_B\left(1+\beta\ A_O\right)$$

Real Op Amp Frequency Response

The closed Loop Amplifier has a lower gain than the Open Loop Amplifier



The closed Loop Amplifier has a higher bandwidth than the Open Loop Amplifier

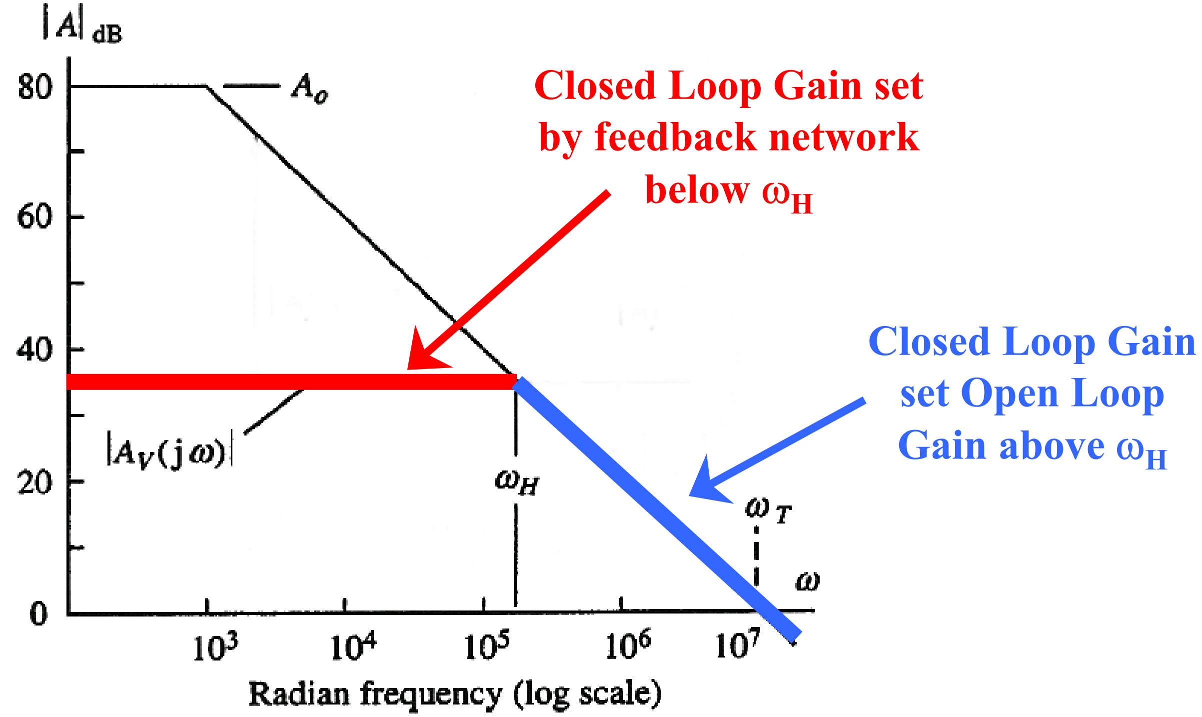
Closed Loop Bandwidth

$$\varpi_H = \varpi_B\left(1+\beta A_O\right) = \frac{\varpi_T}{A_{V,ClosedLoop}\Big|_{@DC}}$$

Closed Loop DC Gain

$$A_{V,ClosedLoop} = \frac{A_{OpenLoop}}{1+\beta\ A_{OpenLoop}}$$

Real Op Amp Frequency Response



$$\left(Gain \quad x \;\; Bandwidth\right)\Big|_{Open \quad Loop} = \left(Gain \quad x \;\; Bandwidth\right)\Big|_{Closed \quad Loop}$$

Example: 741 Op Amp is used as a low pass filter with f_L=10kHz. What is the maximum voltage gain possible for this circuit?

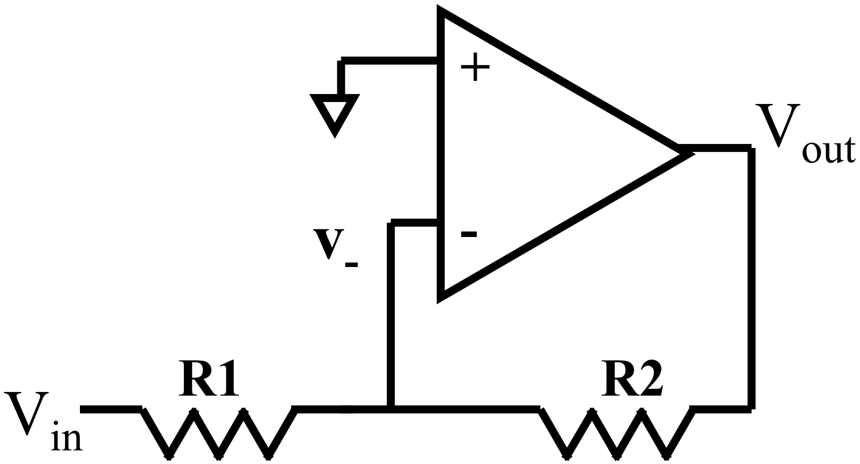
From before, we can write:

$$\left(200{,}000 \quad x \;\; 5\right)\Big|_{Open \quad Loop} = \left(Gain \quad x \; 10{,}000\right)\Big|_{Closed \quad Loop}$$

$$\left(Gain\right)\Big|_{Closed \quad Loop} = 100 \; V\!\!\diagup\!\!V \;\; Maximum$$

Real Op Amp Frequency Response

For the Inverting Configuration:



$$By\ \sup erposition,$$

$$v_- = v_{out}\frac{R_1}{R_1+R_2} + v_{in}\frac{R_2}{R_1+R_2}$$

$$v_- = v_{out}\beta + v_{in}\beta\frac{R_2}{R_1}$$

$$but,$$

$$v_{out} = -v_- A_{V,OpenLoop}$$

$$so,$$

$$-\frac{v_{out}}{A_{V,OpenLoop}} = v_{out}\beta + v_{in}\beta\frac{R_2}{R_1}$$

$$A_{V,ClosedLoop} = \frac{v_{out}}{v_{in}} = \frac{A_{V,OpenLoop}\beta}{1+A_{V,OpenLoop}\beta}\left(-\frac{R_2}{R_1}\right)$$

Real Op Amp Frequency Response

Inserting the frequency dependent open loop gain:

$$A_{V,ClosedLoop} = \frac{A_{V,OpenLoop}\beta}{1+A_{V,OpenLoop}\beta}\left(-\frac{R_2}{R_1}\right)$$

$$A_{V,ClosedLoop} = \frac{\left(\frac{A_O\varpi_B}{s+\varpi_B}\right)\beta}{1+\left(\frac{A_O\varpi_B}{s+\varpi_B}\right)\beta}\left(-\frac{R_2}{R_1}\right) = \frac{A_O\varpi_B\beta}{s+\varpi_B+A_O\varpi_B\beta}\left(-\frac{R_2}{R_1}\right)$$

$$A_{V,ClosedLoop} = \frac{A_O\varpi_B\beta}{s+\varpi_B\left(1+A_O\beta\right)}\left(-\frac{R_2}{R_1}\right) = \frac{\frac{A_O\varpi_B\beta}{\varpi_B\left(1+A_O\beta\right)}}{\frac{s+\varpi_B\left(1+A_O\beta\right)}{\varpi_B\left(1+A_O\beta\right)}}\left(-\frac{R_2}{R_1}\right)$$

$$A_{V,ClosedLoop} = \left(\frac{\frac{A_O\beta}{\left(1+A_O\beta\right)}\left(-\frac{R_2}{R_1}\right)}{\frac{s}{\varpi_B\left(1+A_O\beta\right)}+1}\right)$$

Real Op Amp Frequency Response

$$A_{V,ClosedLoop} = \left(\frac{\dfrac{A_O \beta}{(1 + A_O \beta)} \left(-\dfrac{R_2}{R_1} \right)}{\dfrac{s}{\varpi_B (1 + A_O \beta)} + 1} \right)$$

$$A_{V,ClosedLoop} = \left(\frac{1}{1 + \dfrac{s}{\varpi_B (1 + A_O \beta)}} \right) A_{V,ClosedLoop}\Big|_{@\,DC}$$

Closed Loop Bandwidth

$$\varpi_H = \varpi_B (1 + \beta A_O) = \frac{\varpi_T}{A_{V,ClosedLoop}\Big|_{@\,DC}}$$

Closed Loop DC Gain

$$A_{V,ClosedLoop} = \frac{A_{V,OpenLoop}\beta}{1 + A_{V,OpenLoop}\beta} \left(-\frac{R_2}{R_1} \right)$$

The frequency behavior is the same as for the the Non-Inverting case!