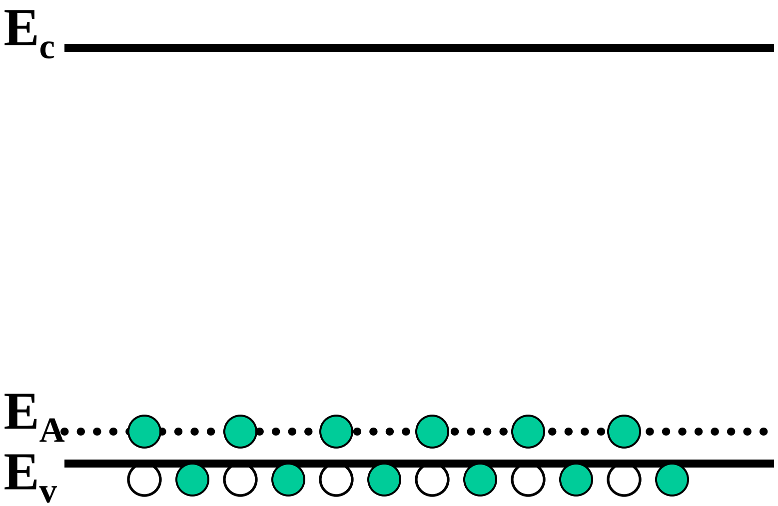
Lecture 6

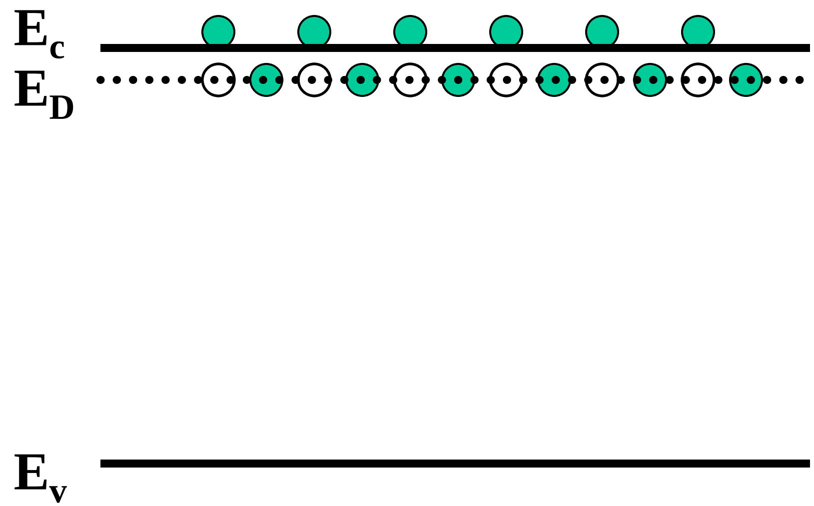
Partial Ionization, Intrinsic Energy and Parameter Relationships

Reading:

(Cont'd) Pierret 2.5-2.6

Partial Ionization Case





$$N_A^- = \frac{N_A}{1+g_A e^{(E_A-E_f)/kT}}$$

$$N_D^+ = \frac{N_D}{1+g_D e^{(E_f-E_D)/kT}}$$

g_A = 4 for Si, GaAs, Ge and most semiconductors

For 10^{14} cm^{-3} B in Si:

$$N_A^- = 0.9998 N_A$$

For 10^{17} cm^{-3} B in Si:

$$N_A^- = 0.88 N_A$$

g_D = 2 for Si, GaAs, Ge and most semiconductors

For 10^{17} cm^{-3} P in Si:

$$N_D^+ = 0.94 N_D$$

Charge Neutrality

Partial Ionization Case

$$\left(p - N_A^-\right) = \left(n - N_d^+\right)$$

$$p - \frac{N_A}{1 + g_A\, e^{(E_A - E_f)/kT}} = n - \frac{N_D}{1 + g_D\, e^{(E_f - E_D)/kT}}$$

$$N_v e^{(E_v - E_f)/kT} - \frac{N_A}{1 + g_A\, e^{(E_A - E_f)/kT}} = \ldots$$

$$\ldots\ N_c e^{(E_f - E_c)/kT} - \frac{N_D}{1 + g_D\, e^{(E_f - E_D)/kT}}$$

Charge Neutrality

Partial Ionization and Non-Degenerate Case

Constants and Information Given		
Ec=	1.12	
Ev=	0	
Nd=	1.00E+17	
gd=	2	Spin of either state or no electron
Ed=	1.066	eV above Ev = (Ec-Eb)
Na=	9.50E+16	
ga=	4	Spin of either state, no electron 2valence bands
Ea=	0.072	eV above Ev
Ef=	0.893168193	<--- This is what you guesstimate
Nc=	3.19568E+19	
Nv=	1.81747E+19	
me=	1.18	
mh=	0.81	
Mc=	1	equivalent minima in conduction band
eps_par=	11.9	
eps_per=	11.9	
kT=	0.025740283	
h=	6.63E-34	
ni=	8.582E+09	
T=	25	C

Equations to Be Solved

(Left Side)=(Right Side)

(Free holes - Ionized Acceptors)=(Free Electrons + Ionized Donors)

$$N_v e^{(E_v-E_f)/kT} - \frac{N_A}{1+g_A\ e^{(E_A-E_f)/kT}} = \ldots$$

$$\ldots\ N_c e^{(E_f-E_c)/kT} - \frac{N_D}{1+g_D\ e^{(E_f-E_D)/kT}}$$

np=	7.37E+19
ni^2=	7.37E+19

Current Solution based on Ef above

Ef=	Free Holes=	Ionized Acceptors=	free electrons=	Ionized Donors=	Left Side=	Right Side =	Left-Right=
0.893168	1.548E+04	9.500E+16	4.758E+15	9.976E+16	-9.500E+16	-9.500E+16	-1.840E+03

History of Guesses

	Ef=	Free Holes=	Ionized Acceptors=	free electrons=	Ionized Donors=	Left Side=	Right Side =	Residual (should Ideally equal 0) Left-Right=	Comment
a	0.56	6472203525	9.5E+16	11380141139	1E+17	-9.5E+16	-1.00E+17	5.00E+15	Initial Guess ~ midgap
b	0.84	122136.1059	9.5E+16	6.03E+14	9.99693E+16	-9.5E+16	-9.94E+16	4.3662E+15	residual reduced no sign change (headed right direction)
c	0.98	530.5667389	9.5E+16	1.39E+17	9.33883E+16	-9.5E+16	4.54E+16	-1.40434E+17	residual increased and sign flip (too high)
d	0.9	11871.72321	9.5E+16	6.20E+15	9.96846E+16	-9.5E+16	-9.35E+16	-1.51962E+15	residual dereased and sign flip from b (still too high)
e	0.87	38078.41965	9.5E+16	1.93429E+15	9.99015E+16	-9.5E+16	-9.7967E+16	2.96716E+15	residual increased sign change from d (d was closer)
f	0.89	17507.95736	9.5E+16	4.20692E+15	9.97859E+16	-9.5E+16	-9.5579E+16	5.78989E+14	residual decreased same sign as e (closer and right direction)
g	0.895	14416.99081	9.5E+16	5.10887E+15	9.97401E+16	-9.5E+16	-9.4631E+16	-3.68745E+14	residual decreased sign changed (closer but overshot the answer)
h	0.893168	15480.36452	9.5E+16	4.75794E+15	9.97579E+16	-9.5E+16	-9.5E+16	-1840	.... Answer to several decimal places (g, two is sufficient ~0.89 eV)

Rho vs e and | Si | F0.5(Ef-Ec) | F0.5(Ev-Ef) | ni vs T | Ni vs T for Si GaN and GaAs

Ready

Charge Neutrality

Partial Ionization and Degenerate Case

$$n = \frac{m_n^* \sqrt{2m_n^*}(kT)^{3/2}}{\pi^2 \hbar^3} \underbrace{\int_0^\infty \frac{\eta^{1/2}}{1+e^{(\eta-\eta_c)}} d\eta}$$

This is known as the Fermi-dirac integral of order 1/2 or, $F_{1/2}(\eta_c)$

$$\left(p - N_A^-\right) = \left(n - N_d^+\right)$$

$$p - \frac{N_A}{1 + g_A e^{(E_A - E_f)/kT}} = n - \frac{N_D}{1 + g_D e^{(E_f - E_D)/kT}}$$

$$N_v \frac{2}{\sqrt{\pi}} F_{1/2}(\eta_v) - \frac{N_A}{1 + g_A e^{(E_A - E_f)/kT}} = \ldots$$

$$\ldots \; N_c \frac{2}{\sqrt{\pi}} F_{1/2}(\eta_c) - \frac{N_D}{1 + g_D e^{(E_f - E_D)/kT}}$$

where $\eta_v = (E_v - E_f)/kT$

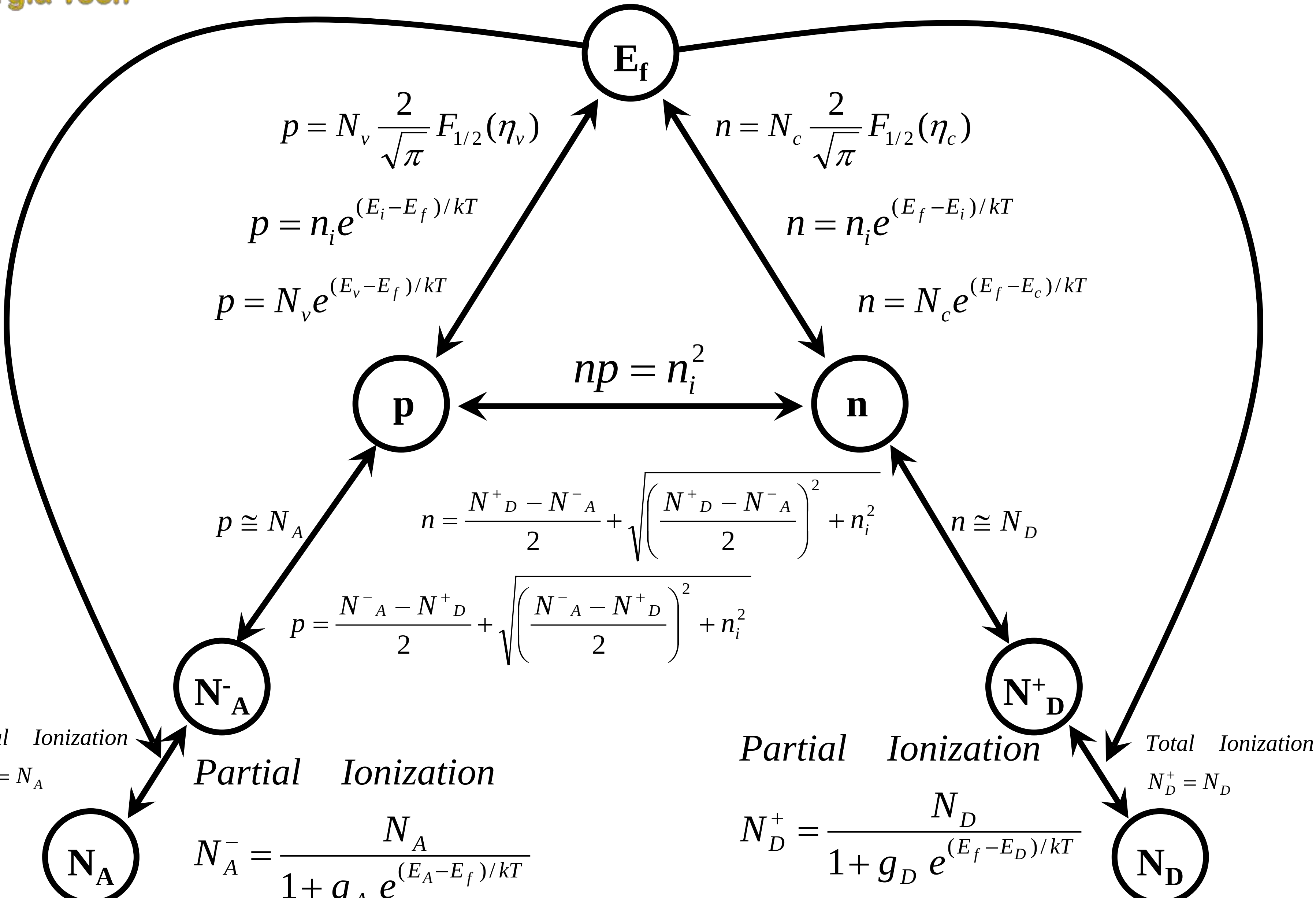
What are the Degeneracy Factors

The degeneracy factors account for the possibility of electrons with different spin, occupying the same energy level (I.E. a true statement of the Pauli Exclusion principle is that no electron with the same quantum numbers (energy and spin) can occupy the same state).

g_D is then =2 in most semiconductors.

g_A is 4 due to the above reason combined with the fact that there are actually 2 valence bands in most semiconductors. Thus, 2 spins x 2 valance bands makes g_A=4

Relationships between Parameters



E_f ↔ p:

$$p = N_v \frac{2}{\sqrt{\pi}} F_{1/2}(\eta_v)$$

$$p = n_i e^{(E_i - E_f)/kT}$$

$$p = N_v e^{(E_v - E_f)/kT}$$

E_f ↔ n:

$$n = N_c \frac{2}{\sqrt{\pi}} F_{1/2}(\eta_c)$$

$$n = n_i e^{(E_f - E_i)/kT}$$

$$n = N_c e^{(E_f - E_c)/kT}$$

p ↔ n:

$$np = n_i^2$$

p ↔ N^-_A:

$$p \cong N_A$$

n ↔ N^+_D:

$$n \cong N_D$$

$$n = \frac{N^+{}_D - N^-{}_A}{2} + \sqrt{\left(\frac{N^+{}_D - N^-{}_A}{2}\right)^2 + n_i^2}$$

$$p = \frac{N^-{}_A - N^+{}_D}{2} + \sqrt{\left(\frac{N^-{}_A - N^+{}_D}{2}\right)^2 + n_i^2}$$

Total Ionization

$$N_A^- = N_A$$

Partial Ionization

$$N_A^- = \frac{N_A}{1 + g_A\, e^{(E_A - E_f)/kT}}$$

Partial Ionization

$$N_D^+ = \frac{N_D}{1 + g_D\, e^{(E_f - E_D)/kT}}$$

Total Ionization

$$N_D^+ = N_D$$

Where is E_i?

Since we started with descriptions of intrinsic materials then it makes sense to reference energies from the intrinsic energy, E_i.

Intrinsic Material:

$$n = N_c e^{(E_f - E_c)/kT} = N_v e^{(E_v - E_f)/kT} = p$$

$$N_c e^{(E_i - E_c)/kT} = N_v e^{(E_v - E_i)/kT}$$

$$E_i = \frac{E_c + E_v}{2} + \frac{kT}{2} \ln\left(\frac{N_v}{N_c}\right)$$

Where is E_i?

Intrinsic Material:

But,

$$\frac{N_v}{N_c} = \left(\frac{m_p^*}{m_n^*}\right)^{3/2}$$

$$E_i = \frac{E_c + E_v}{2} + \frac{3kT}{4}\ln\left(\frac{m_p^*}{m_n^*}\right)$$

Letting E_v=0, this is E_g / 2 or "Midgap"

-0.007 eV for Si @ 300K (0.6% of E_G)

$$E_i = \frac{E_g}{2} + \frac{3kT}{4}\ln\left(\frac{m_p^*}{m_n^*}\right)$$

Where is E_i?

Extrinsic Material:

$$n = n_i e^{(E_f - E_i)/kT} \qquad p = n_i e^{(E_i - E_f)/kT}$$

Solving for (E_f-E_i)

$$E_f - E_i = kT \ln\left(\frac{n}{n_i}\right) = -kT \ln\left(\frac{p}{n_i}\right)$$

$$or \ \ for \ N_D >> N_A \ and \ N_D >> n_i$$

$$E_f - E_i = kT \ln\left(\frac{N_D}{n_i}\right)$$

$$or \ \ for \ N_A >> N_D \ and \ N_A >> n_i$$

$$E_f - E_i = -kT \ln\left(\frac{N_A}{n_i}\right)$$

Where is E_i?

Extrinsic Material:

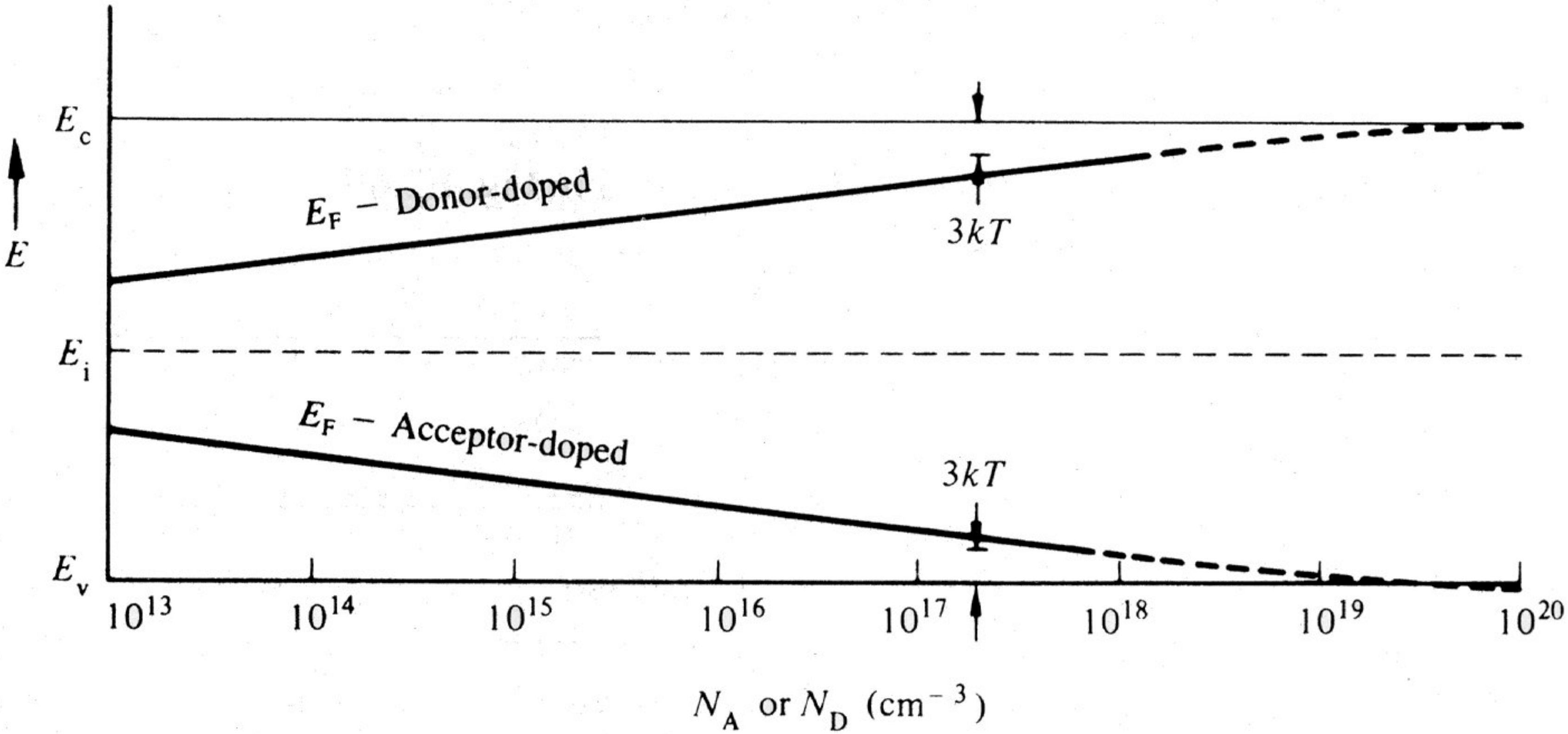


Figure 2.21 Fermi level positioning in Si at 300 K as a function of the doping concentration. The solid E_F lines were established using Eq. (2.38a) for donor-doped material and Eq. (2.38b) for acceptor-doped material ($kT = 0.0259$ eV, and $n_i = 10^{10}/cm^3$).

Note: The fermi-level is pictured here for 2 separate cases: acceptor and donor doped.