Lecture 10

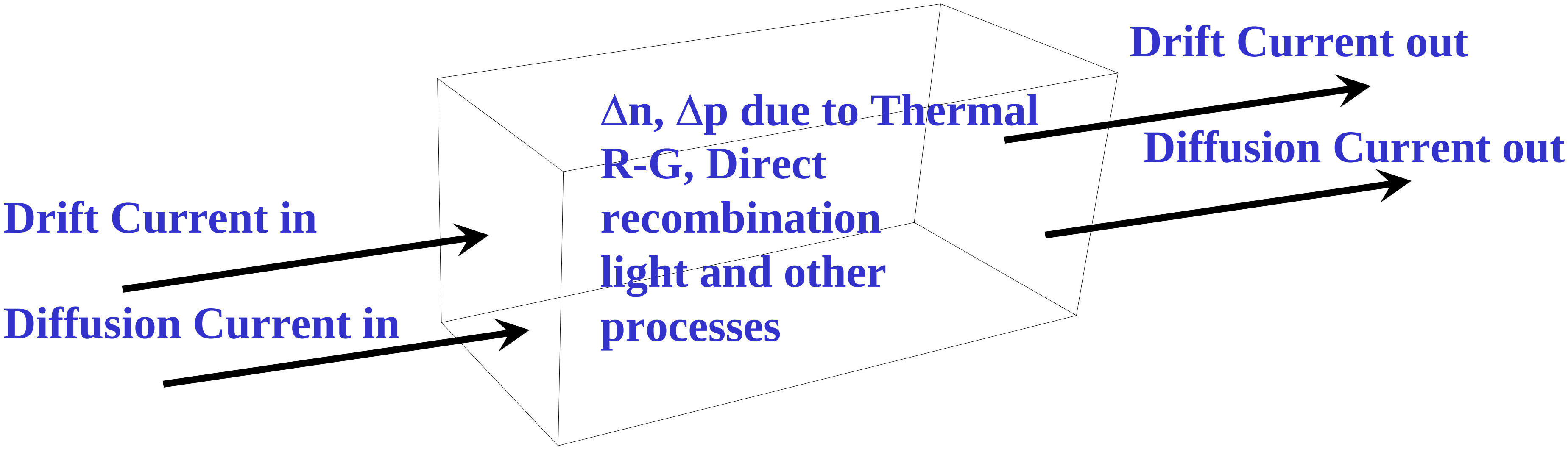
Equations of State, Minority Carrier Diffusion Equation and Quasi-Fermi Levels

Reading:

(Cont'd) Notes and Anderson[2] sections 3.4-3.11 and chapter 4

Ways carrier concentrations can be altered

Continuity Equations



$$\frac{\partial n}{\partial t} = \left.\frac{\partial n}{\partial t}\right|_{Drift} + \left.\frac{\partial n}{\partial t}\right|_{Diffusion} + \left.\frac{\partial n}{\partial t}\right|_{\mathrm{Re}\,combination-Generation} + \left.\frac{\partial n}{\partial t}\right|_{\substack{All\ other\ processes \\ such\ as\ light,\ etc...}}$$

$$\frac{\partial p}{\partial t} = \left.\frac{\partial p}{\partial t}\right|_{Drift} + \left.\frac{\partial p}{\partial t}\right|_{Diffusion} + \left.\frac{\partial p}{\partial t}\right|_{\mathrm{Re}\,combination-Generation} + \left.\frac{\partial p}{\partial t}\right|_{\substack{All\ other\ processes \\ such\ as\ light,\ etc...}}$$

There must be spatial and time continuity in the carrier concentrations

Ways carrier concentrations can be altered

Continuity Equations

$$\left.\frac{\partial n}{\partial t}\right|_{Drift} + \left.\frac{\partial n}{\partial t}\right|_{Diffusion} = \frac{1}{q}\left(\frac{\partial J_{Nx}}{\partial x} + \frac{\partial J_{Ny}}{\partial y} + \frac{\partial J_{Nz}}{\partial z}\right) = \frac{1}{q}\nabla \cdot J_N$$

$$\left.\frac{\partial p}{\partial t}\right|_{Drift} + \left.\frac{\partial p}{\partial t}\right|_{Diffusion} = -\frac{1}{q}\left(\frac{\partial J_{Px}}{\partial x} + \frac{\partial J_{Py}}{\partial y} + \frac{\partial J_{Pz}}{\partial z}\right) = -\frac{1}{q}\nabla \cdot J_P$$

Divergence in the current

$$\frac{\partial n}{\partial t} = \frac{1}{q}\nabla \cdot J_N + \left.\frac{\partial n}{\partial t}\right|_{\mathrm{Re}\,combination-Generation} + \left.\frac{\partial n}{\partial t}\right|_{\substack{All\ other\ processes \\ such\ as\ light,\ etc...}}$$

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\nabla \cdot J_P + \left.\frac{\partial p}{\partial t}\right|_{\mathrm{Re}\,combination-Generation} + \left.\frac{\partial p}{\partial t}\right|_{\substack{All\ other\ processes \\ such\ as\ light,\ etc...}}$$

Continuity Equations: Special Case known as "Minority Carrier Diffusion Equation"

Continuity Equation

Minority Carrier Diffusion Equation

Simplifying Assumptions:

1) One dimensional case. We will use "x".

2) We will only consider minority carriers

3) Electric field is approximately zero in regions subject to analysis.

4) The minority carrier concentrations IN EQUILIBRIUM are not a function of position.

5) Low-level injection conditions apply.

6) SRH recombination-generation is the main recombination-generation mechanism.

7) The only "other" mechanism is photogeneration.

Continuity Equations: Special Case known as "Minority Carrier Diffusion Equation"

Because of (3) - no electric field …

$$Since\ E = 0, \quad ...$$

$$J_n = \cancelto{0}{J_n|_{Drift}} + J_n|_{Diffusion} = \cancelto{0}{q\mu_n nE} + qD_n \nabla n$$

$$J_n = J_n|_{Diffusion} = qD_n \frac{\partial n}{\partial x}$$

$$\frac{1}{q}\nabla \cdot J_N = \frac{1}{q}\frac{\partial J_N}{\partial x} = D_n \frac{\partial^2 n}{\partial x^2} = D_n \frac{\partial^2 (n_o + \Delta n)}{\partial x^2} = D_n \frac{\partial^2 (\Delta n)}{\partial x^2}$$

Continuity Equations: Special Case known as "Minority Carrier Diffusion Equation"

Because of (5) - low level injection …

$$\left.\frac{\partial n}{\partial t}\right|_{Recombination-Generation} = -\frac{\Delta n}{\tau_n}$$

Because of (7) - no other process …

$$\left.\frac{\partial n}{\partial t}\right|_{\substack{All\ other\ processes \\ such\ as\ light,\ etc...}} = G_L$$

Continuity Equations: Special Case known as “Minority Carrier Diffusion Equation”

Finally …

$$\frac{\partial n}{\partial t} = \frac{\partial (n_o + \Delta n)}{\partial t} = \frac{\partial (\Delta n)}{\partial t}$$

$$\frac{\partial n}{\partial t} = \left.\frac{\partial n}{\partial t}\right|_{Drift} + \left.\frac{\partial n}{\partial t}\right|_{Diffusion} + \left.\frac{\partial n}{\partial t}\right|_{\mathrm{Re}\,combination-Generation} + \left.\frac{\partial n}{\partial t}\right|_{\substack{All\ other\ processes \\ such\ as\ light,\ etc...}}$$

$$\boxed{\begin{gathered} \frac{\partial (\Delta n_p)}{\partial t} = D_N \frac{\partial^2 (\Delta n_p)}{\partial x^2} - \frac{(\Delta n_p)}{\tau_n} + G_L \\ or \\ \frac{\partial (\Delta p_n)}{\partial t} = D_P \frac{\partial^2 (\Delta p_n)}{\partial x^2} - \frac{(\Delta p_n)}{\tau_p} + G_L \end{gathered}}$$

Minority Carrier Diffusion Equations

Continuity Equations: Special Case known as "Minority Carrier Diffusion Equation" - Further simplifications

Steady State …

$$\frac{\partial(\Delta n_p)}{\partial t} \rightarrow 0 \quad and \quad \frac{\partial(\Delta p_n)}{\partial t} \rightarrow 0$$

No minority carrier diffusion gradient …

$$D_N \frac{\partial^2(\Delta n_p)}{\partial x^2} \rightarrow 0 \quad and \quad D_P \frac{\partial^2(\Delta p_n)}{\partial x^2} \rightarrow 0$$

No SRH recombination-generation …

$$\frac{\Delta n_p}{\tau_n} = 0 \quad and \quad \frac{\Delta p_n}{\tau_p} = 0$$

No Light …

$$G_L \rightarrow 0$$

Solutions to the Minority Carrier Diffusion Equation

Consider a semi-infinite p-type silicon sample with $N_A=10^{15}$ cm^{-3} constantly illuminated by light absorbed in a very thin region of the material creating a steady state excess of 10^{13} cm^{-3} minority carriers. What is the minority carrier distribution in the region x>0?

Light absorbed
in a thin skin

$$\cancelto{0}{\frac{\partial(\Delta n_p)}{\partial t}} = D_N \frac{\partial^2(\Delta n_p)}{\partial x^2} - \frac{(\Delta n_p)}{\tau_n} + \cancelto{0}{G_L}$$

$$D_N \frac{\partial^2(\Delta n_p)}{\partial x^2} = \frac{(\Delta n_p)}{\tau_n}$$

Solutions to the Minority Carrier Diffusion Equation

General Solution …

$$\Delta n_p(x) = Ae^{(-x/L_N)} + Be^{(+x/L_N)} \quad where \quad L_N \equiv \sqrt{D_n \tau_n}$$

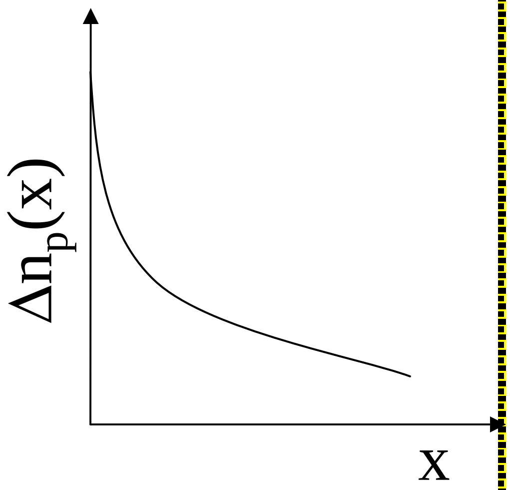
L_N is the "diffusion length" the average distance a minority carrier can move before recombining with a majority carrier.

Boundary Condition …

$$\Delta n_p(x=0) = 10^{13}\ cm^{-3} = A + B$$

$$\Delta n_p(x=\infty) = 0 = A(0) + Be^{(+\infty/L_N)}$$

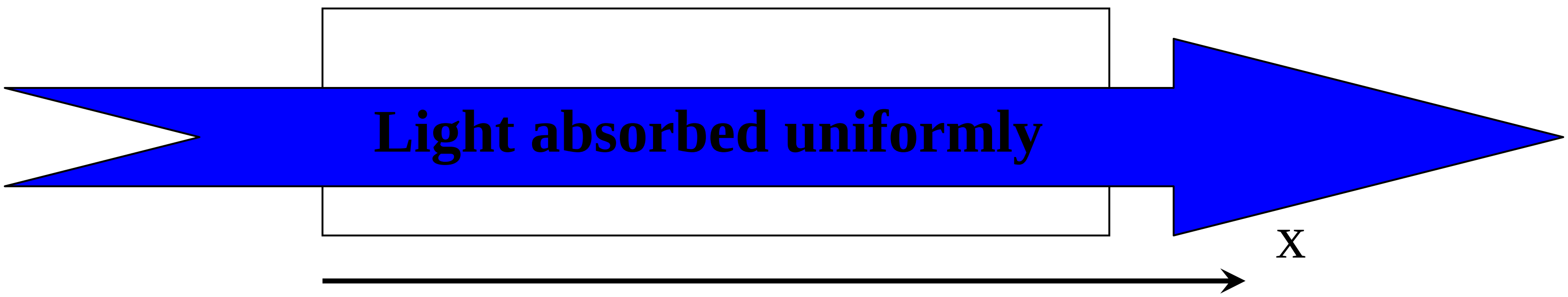
$$\Rightarrow B = 0$$



$$\boxed{\Delta n_p(x) = 10^{13}\ e^{(-x/L_N)} \quad cm^{-3}}$$

Solutions to the Minority Carrier Diffusion Equation

Consider a p-type silicon sample with $N_A=10^{15}$ cm^{-3} and minority carrier lifetime τ=1 uS constantly illuminated by light absorbed uniformly throughout the material creating an excess 10^{13} cm^{-3} minority carriers <u>per second</u>. The light has been on for a very long time. At time t=0, the light is shut off. What is the minority carrier distribution in for t<0?

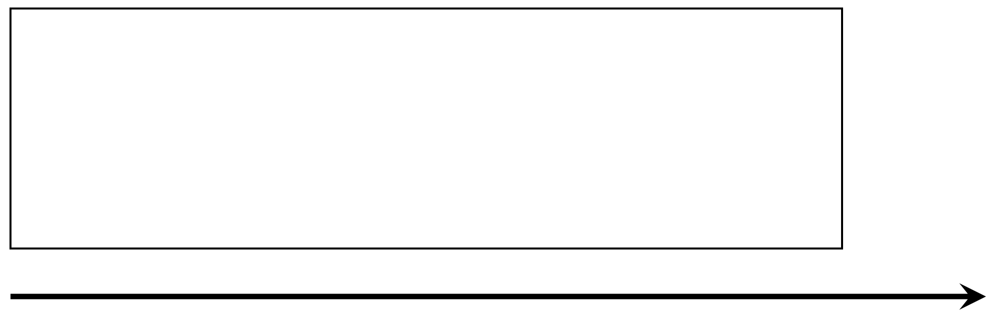


$$\cancelto{0}{\frac{\partial(\Delta n_p)}{\partial t}} = D_N \cancelto{0}{\frac{\partial^2(\Delta n_p)}{\partial x^2}} - \frac{(\Delta n_p)}{\tau_n} + G_L$$

$$\Delta n_p(all\ \ x, t<0) = G_L \tau_n = 10^7\ cm^{-3}$$

Solutions to the Minority Carrier Diffusion Equation

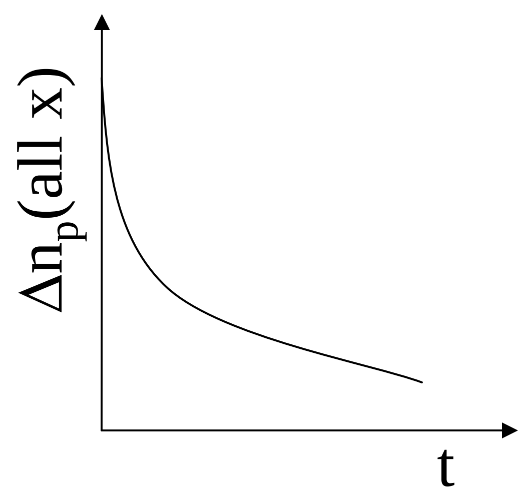
In the previous example: What is the minority carrier distribution in for t>0?



$$\frac{\partial(\Delta n_p)}{\partial t} = D_N \cancelto{0}{\frac{\partial^2(\Delta n_p)}{\partial x^2}} - \frac{(\Delta n_p)}{\tau_n} + \cancelto{0}{G_L}$$

$$\Delta n_p(t) = \left[\Delta n_p(t=0)\right] e^{\left(-t/\tau_n\right)}$$

$$\boxed{\Delta n_p(t) = 10^7 e^{\left(-t/1e{-6}\right)}}$$



Quasi - Fermi Levels

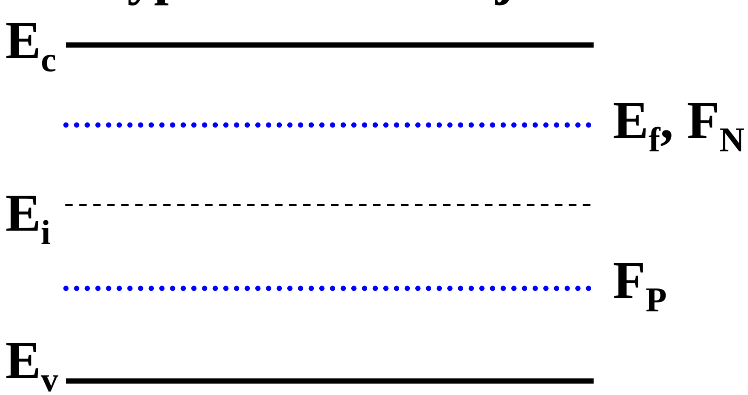
Equilibrium:

$$n_o = n_i e^{\left((E_f - E_i) / kT \right)} \quad and \quad p_o = n_i e^{\left((E_i - E_f) / kT \right)}$$

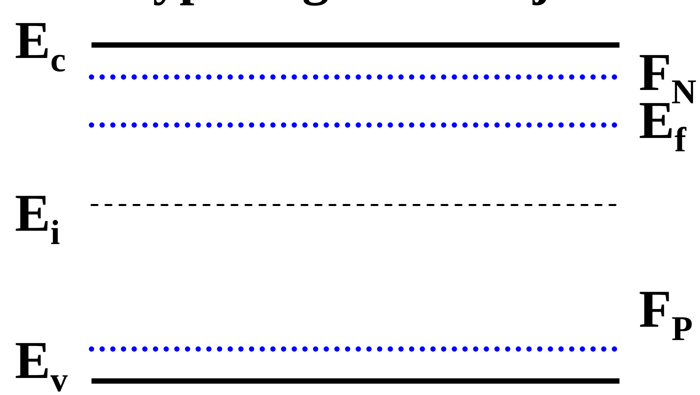
Non-equilibrium:

$$n = n_i e^{\left((F_N - E_i) / kT \right)} \quad and \quad p = n_i e^{\left((E_i - F_P) / kT \right)}$$

N-type Low level injection



N-type High level injection



Quasi - Fermi Levels

$$p = n_i e^{\left((E_i - F_P) / kT \right)}$$

$$\nabla p = \frac{n_i}{kT} e^{\left((E_i - F_P) / kT \right)} \left(\nabla E_i - \nabla F_P \right)$$

$$\nabla p = \left(\frac{qp}{kT} \right) E - \left(\frac{p}{kT} \right) \nabla F_P$$

$$J_p = q\mu_p pE - qD_p \nabla p$$

$$J_p = q\left(\mu_p - \frac{qD_p}{kT} \right) pE + \left(\frac{qD_p}{kT} \right) p \nabla F_P$$

$$but... \quad \mu_p = \frac{qD_P}{kT} \quad leading\ to...$$

$$J_p = \mu_p p \nabla F_P \quad and\ similarly \quad J_N = \mu_n n \nabla F_N$$

If there is a change in quasi-fermi levels, current flows!