Lecture 11

P-N Junction Diodes: Part 1
How do they work? (postponing the math)

Reading:
(Cont’d) Notes and Anderson² sections Part 2 preface
and sections 5.0-5.3
A p-n junction diode is made by forming a p-type region of material directly next to a n-type region.
Our First Device: p-n Junction Diode

In regions far away from the “junction” the band diagram looks like:

- $E_c$
- $E_i$
- $E_f$
- $E_v$
Our First Device: p-n Junction Diode

But when the device has no external applied forces, no current can flow. Thus, the fermi-level must be flat!

We can then fill in the junction region of the band diagram as:

\[ \begin{align*}
E_c & \\
E_i & \\
E_f & \\
E_v & \\
\end{align*} \]

or…
Our First Device: p-n Junction Diode
Our First Device: p-n Junction Diode

Electrostatic Potential,
\[ V = -(1/q)(E_c - E_{ref}) \]

\(-qV_{BI}\) or the “built in potential”
Our First Device: p-n Junction Diode

Electrostatic Potential,

\[ V = -\frac{1}{q}(E_c - E_{ref}) \]

Electric Field

\[ E = -\frac{dV}{dx} \]
Our First Device: p-n Junction Diode

Poisson’s Equation:

\[ \nabla \cdot E = \frac{\rho}{K_s \varepsilon_o} \quad \text{or in 1D,} \quad \frac{dE}{dx} = \frac{\rho}{K_s \varepsilon_o} \]

- Electric Field
- Charge Density (NOT resistivity)
- Permittivity of free space
- Relative Permittivity of Semiconductor
  (previously referred to as \( \varepsilon_R \))

\[ \rho = q( p - n + N_D - N_A ) \]
Our First Device: p-n Junction Diode

Electric Field, $E = -\frac{dV}{dx}$

$$\rho = K_s \varepsilon_o \frac{dE}{dx}$$
Our First Device: p-n Junction Diode

Energy

-1/q

Potential

-dV/dx

Electric Field

\( K_s \varepsilon_o \frac{dE}{dx} \)

Charge Density
P-N Junction Diodes: Part 2

How do they work? (A little bit of math)
Movement of electrons and holes when forming the junction

- Circles are charges free to move (electrons and holes)
- Squares are charges NOT free to move (ionized donor or acceptor atoms)

Electron diffusion

Hole diffusion

Local region of negative charge due to imbalance in hole-acceptor concentrations

Local region of positive charge due to imbalance in electron-donor concentrations

Space Charge or Depletion Region
Movement of electrons and holes when forming the junction

\[ E = -\frac{dV}{dx} \]

\[-E dx = dV \]

\[-\int_{-x_p}^{x_n} E dx = \int_{V(-x_p)}^{V(x_n)} dV = V(x_n) - V(-x_p) = V_{bi} \]

but...

\[ J_N = q\mu_n nE + qD_N \frac{dn}{dx} = 0 \]

No net current flow in equilibrium

\[ E = -\frac{D_N}{\mu_n n} \frac{dn}{dx} = -kT \frac{dn}{n} \]

thus...

\[ V_{bi} = -\int_{-x_p}^{x_n} E dx = \frac{kT}{q} \int_{n(-x_p)}^{n(x_n)} \frac{dn}{n} dx = \frac{kT}{q} \ln \left[ \frac{n(x_n)}{n(-x_p)} \right] \]
Movement of electrons and holes when forming the junction

\[ V_{bi} = \frac{kT}{q} \ln \left( \frac{n(x_n)}{n(-x_p)} \right) = \frac{kT}{q} \ln \left( \frac{N_D}{n_i^2 \sqrt{N_A}} \right) \]

\[ V_{bi} = kT \ln \left( \frac{N_A N_D}{n_i^2} \right) \]

For \( N_A = N_D = 10^{15}/\text{cm}^3 \) in silicon at room temperature,

\[ V_{bi} \sim 0.6 \text{ V}^* \]

For a non-degenerate semiconductor, \(|-qV_{bi}| < |E_g|\)

*Note to those familiar with a diode turn on voltage: This is not the diode turn on voltage! This is the voltage required to reach a flat band diagram and sets an upper limit (typically an overestimate) for the voltage that can be applied to a diode before it burns itself up.
Movement of electrons and holes when forming the junction

Depletion Region Approximation

Depletion Region Approximation states that approximately no free carriers exist in the space charge region and no net charge exists outside of the depletion region (known as the quasi-neutral region). Thus,

$$\frac{dE}{dx} = \frac{\rho}{K_S \varepsilon_o} = \frac{q}{K_S \varepsilon_o} (p - n + N_D - N_A) = 0 \quad \text{within the quasi-neutral region}$$

becomes...

$$\frac{dE}{dx} = \frac{q}{K_S \varepsilon_o} (N_D - N_A) \quad \text{within the space charge region}$$
Movement of electrons and holes when forming the junction

Depletion Region Approximation: Step Junction Solution

\[
\rho = \begin{cases} 
  -qN_A & \text{for } -x_p \leq x \leq 0 \\
  qN_D & \text{for } 0 \leq x \leq x_n \\
  0 & \text{for } x \leq -x_p \text{ and } x \geq x_n 
\end{cases}
\]

thus,

\[
\frac{dE}{dx} = \begin{cases} 
  -\frac{qN_A}{K_S \varepsilon_o} & \text{for } -x_p \leq x \leq 0 \\
  \frac{qN_D}{K_S \varepsilon_o} & \text{for } 0 \leq x \leq x_n \\
  0 & \text{for } x \leq -x_p \text{ and } x \geq x_n 
\end{cases}
\]

Where we have used:

\[
\frac{dE}{dx} = \frac{\rho}{K_S \varepsilon_o}
\]
Movement of electrons and holes when forming the junction

Depletion Region Approximation: Step Junction Solution

\[ \int_{0}^{E(x)} dE' = \int_{-x_p}^{x} -\frac{qN_A}{K_S \varepsilon_o} dx' \quad \text{for} \quad -x_p \leq x \leq 0 \]

\[ E(x) = \frac{-qN_A}{K_S \varepsilon_o} (x + x_p) \quad \text{for} \quad -x_p \leq x \leq 0 \]

and

\[ \int_{E(x)}^{0} dE' = \int_{x}^{x_n} \frac{qN_D}{K_S \varepsilon_o} dx' \quad \text{for} \quad 0 \leq x \leq x_n \]

\[ E(x) = \frac{-qN_D}{K_S \varepsilon_o} (x_n - x) \quad \text{for} \quad 0 \leq x \leq x_n \]

Since \( E(x=0^-) = E(x=0^+) \)

\[ N_A x_p = N_D x_n \]
Movement of electrons and holes when forming the junction

Depletion Region Approximation: Step Junction Solution

\[ E = -\frac{dV}{dx} \]

\[
\frac{dV}{dx} = \begin{cases} 
\frac{qN_A}{K_S \varepsilon_o} (x_p + x) & \text{for } -x_p \leq x \leq 0 \\
\frac{qN_D}{K_S \varepsilon_o} (x_n - x) & \text{for } 0 \leq x \leq x_n 
\end{cases}
\]

or,

\[
\int_0^{V(x)} dV' = \int_{-x_p}^x \frac{qN_A}{K_S \varepsilon_o} (x_p + x') dx' \quad \text{for } -x_p \leq x \leq 0
\]

\[
\int_{V(x)}^{V_{bi}} dV' = \int_x^{x_n} \frac{qN_D}{K_S \varepsilon_o} (x_n - x') dx' \quad \text{for } 0 \leq x \leq x_n
\]

\[
V(x) = \begin{cases} 
-\frac{qN_A}{2K_S \varepsilon_o} (x_p + x)^2 & \text{for } -x_p \leq x \leq 0 \\
V_{bi} - \frac{qN_D}{2K_S \varepsilon_o} (x_n - x)^2 & \text{for } 0 \leq x \leq x_n
\end{cases}
\]
Movement of electrons and holes when forming the junction

Depletion Region Approximation: Step Junction Solution

At \( x=0 \),

\[
\frac{qN_A}{2K_SE_o}(x_p)^2 = V_{bi} - \frac{qN_D}{2K_SE_o}(x_n)^2
\]

\[
U \sin \theta, \quad x_p = \frac{x_n N_D}{N_A}
\]

\[
x_n = \sqrt{\frac{2K_SE_o}{q N_D (N_A + N_D)} V_{bi}} \quad \text{and} \quad x_p = \sqrt{\frac{2K_SE_o}{q N_A (N_A + N_D)} V_{bi}}
\]

\[
W = x_p + x_n = \sqrt{\frac{2K_SE_o (N_A + N_D)}{q N_A N_D} V_{bi}}
\]
Movement of electrons and holes when forming the junction

Depletion Region Approximation: Step Junction Solution

Negligible voltage drop (low-level injection)

Negligible voltage drop (ohmic contact)

$V_A = 0$: No Bias

$V_A < 0$: Reverse Bias

$V_A > 0$: Forward Bias

$V_{bi}$
Movement of electrons and holes when forming the junction

Depletion Region Approximation: Step Junction Solution

Thus, only the boundary conditions change resulting in direct replacement of $V_{bi}$ with $(V_{bi} - V_A)$

$$x_n = \sqrt{\frac{2K_S}{q} \frac{N_A}{N_D(N_A + N_D)}(V_{bi} - V_A)} \quad \text{and} \quad x_p = \sqrt{\frac{2K_S}{q} \frac{N_D}{N_A(N_A + N_D)}(V_{bi} - V_A)}$$

$$W = x_p + x_n = \sqrt{\frac{2K_S}{q} \frac{(N_A + N_D)}{N_A N_D}(V_{bi} - V_A)}$$
Movement of electrons and holes when forming the junction

Step Junction Solution: What does it mean?

Consider a $p^+$ -n junction (heavily doped p-side, normal or lightly doped n side).
Movement of electrons and holes when forming the junction

Step Junction Solution: What does it mean?

Fermi-level only
applies to equilibrium
(no current flowing)

\[ E_{fp} - E_{fn} = -qV_A \]

(a) Equilibrium \((V_A = 0)\)

Majority carrier
Quasi-fermi levels

(b) Forward bias \((V_A > 0)\)

(c) Reverse bias \((V_A < 0)\)

ECE 3080 - Dr. Alan Doolittle
P-N Junction Diodes: Part 3
Current Flowing through a Diode
P-n Junction I-V Characteristics

In Equilibrium, the Total current balances due to the sum of the individual components

(a) Equilibrium \((V_A = 0)\)
Current flow is proportional to $e^{(V_a/V_{ref})}$ due to the exponential decay of carriers into the majority carrier bands.

Current flow is dominated by majority carriers flowing across the junction and becoming minority carriers.

Electron Diffusion Current

Hole Diffusion Current

(b) Forward bias ($V_A > 0$)
P-n Junction I-V Characteristics

Current flow is constant due to thermally generated carriers swept out by E-fields in the depletion region.

Electron Drift Current $I_N$

Electron Diffusion Current negligible due to large energy barrier.

Hole Diffusion Current negligible due to large energy barrier.

Current flow is dominated by minority carriers flowing across the junction and becoming majority carriers.

(c) Reverse bias ($V_A < 0$)
P-n Junction I-V Characteristics

Where does the reverse bias current come from? Generation near the depletion region edges “replenishes” the current source.
P-n Junction I-V Characteristics
Putting it all together

Reverse Bias: Current flow is constant due to thermally generated carriers swept out by E-fields in the depletion region

Forward Bias: Current flow is proportional to $e^{(V_a/V_{ref})}$ due to the exponential decay of carriers into the majority carrier bands

Current flow is zero at no applied voltage

$I = I_o(e^{V_a/V_{ref}} - 1)$
P-N Junction Diodes: Part 3

Quantitative Analysis (Math, math and more math)
Quantitative p-n Diode Solution

Assumptions:
1) steady state conditions
2) non-degenerate doping
3) one-dimensional analysis
4) low-level injection
5) no light ($G_L = 0$)

Current equations:
$J = J_p(x) + J_n(x)$

$J_n = q \mu_n n E + qD_n (dn/dx)$

$J_p = q \mu_p p E - qD_p (dp/dx)$
Quantitative p-n Diode Solution

**p-type**

\[ E = 0 \]

Depletion Region

\[ E \neq 0 \]

**n-type**

\[ E = 0 \]

Application of the Minority Carrier Diffusion Equation

Since electric fields exist in the depletion region, the minority carrier diffusion equation does not apply here.

\[
\frac{\partial(\Delta n_p)}{\partial t} = D_N \frac{\partial^2 (\Delta n_p)}{\partial x^2} - \frac{\Delta n_p}{\tau_n} + G_L
\]

\[
0 = D_N \frac{\partial^2 (\Delta n_p)}{\partial x^2} - \frac{\Delta n_p}{\tau_n}
\]

Boundary Condition:

\[ \Delta n_p (x \to -\infty) = 0 \]

\[ \Delta n_p (x = -x_p) = ? \]

Boundary Condition:

\[ \Delta p_n (x = x_n) = ? \]

Boundary Condition:

\[ \Delta p_n (x \to \infty) = 0 \]
Quantitative p-n Diode Solution

<table>
<thead>
<tr>
<th>p-type</th>
<th>Depletion Region</th>
<th>n-type</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E = 0$</td>
<td>$E \neq 0$</td>
<td>$E = 0$</td>
</tr>
</tbody>
</table>

Application of the Minority Carrier Diffusion Equation

**Boundary Condition:**

1. For p-type: $\Delta n_p (x = -x_p) = ?$
2. For n-type: $\Delta p_n (x = x_n) = ?$

- $n = n_i e^{(F_n - E_i)/kT}$ and $p = n_i e^{(E_i - F_p)/kT}$

- $n_p (x = -x_p) p_p (x = -x_p) = n_i^2 e^{(F_N - F_p)/kT}$

- $n_p (x = -x_p) p_p (x = -x_p) = n_p (x = -x_p) N_A = n_i^2 e^{qV_A/kT}$

- $n_p (x = -x_p) = \frac{n_i^2}{N_A} e^{qV_A/kT}$

- $\Delta n_p (x = -x_p) = \frac{n_i^2}{N_A} e^{qV_A/kT} - n_o$

- $\Delta n_p (x = -x_p) = \frac{n_i^2}{N_A} \left( e^{qV_A/kT} - 1 \right)$ and similarly at $x = x_n$, $\Delta p_n (x = x_n) = \frac{n_i^2}{N_D} \left( e^{qV_A/kT} - 1 \right)$
Quantitative p-n Diode Solution

- **p-type**
  
  \[ E = 0 \]

- **Depletion Region**
  
  \[ E \neq 0 \]

- **n-type**
  
  \[ E = 0 \]

**Application of the Minority Carrier Diffusion Equation**

**Boundary Condition:**

\[ \Delta n_p (x = -x_p) = ? \]

\[ n = n_i e^{(F_n - E_i)/kT} \quad \text{and} \quad p = n_i e^{(E_i - F_p)/kT} \]

\[ n_p (x = -x_p) p_p (x = -x_p) = n_i^2 e^{(F_N - F_p)/kT} \]

\[ n_p (x = -x_p) p_p (x = -x_p) = n_p (x = -x_p) N_A = n_i^2 e^{qV_A/kT} \]

\[ n_p (x = -x_p) = \frac{n_i^2}{N_A} e^{qV_A/kT} \]

\[ \Delta n_p (x = -x_p) = \frac{n_i^2}{N_A} e^{qV_A/kT} - n_o \]

\[ \Delta n_p (x = -x_p) = \frac{n_i^2}{N_A} \left( e^{qV_A/kT} - 1 \right) \quad \text{and similarly} \quad \Delta p_n (x = x_n) = \frac{n_i^2}{N_D} \left( e^{qV_A/kT} - 1 \right) \]
Quantitative p-n Diode Solution

p-type
$E = 0$

Depletion Region
$E \neq 0$

n-type
$E = 0$

Application of the Current Continuity Equation

\[
J_n = q \left( \mu_n n E + D_n \frac{dn}{dx} \right) = qD_n \frac{d(n_o + \Delta n_p)}{dx} = qD_n \frac{d\Delta n_p}{dx}
\]

\[
J_p = q \left( \mu_p p E - D_p \frac{dp}{dx} \right) = -qD_p \frac{d(p_o + \Delta p_n)}{dx} = -qD_p \frac{d\Delta p_n}{dx}
\]
Quantitative p-n Diode Solution

p-type  
$E = 0$

Depletion Region  
$E \neq 0$

n-type  
$E = 0$

Application of the Current Continuity Equation: Depletion Region

\[
\frac{\partial n}{\partial t} = \frac{1}{q} \nabla \cdot J_n + \frac{\partial n}{\partial t} \left| \text{Recombination--Generation} \right| + \frac{\partial n}{\partial t} \left| \text{All other processes such as light, etc...} \right|
\]

\[
0 = \frac{1}{q} \nabla \cdot J_n
\]

\[
0 = \frac{1}{q} \frac{\partial J_n}{\partial x}
\]

\[
\frac{\partial p}{\partial t} = -\frac{1}{q} \nabla \cdot J_p + \frac{\partial p}{\partial t} \left| \text{Recombination--Generation} \right| + \frac{\partial p}{\partial t} \left| \text{All other processes such as light, etc...} \right|
\]

\[
0 = -\frac{1}{q} \nabla \cdot J_p
\]

\[
0 = -\frac{1}{q} \frac{\partial J_p}{\partial x}
\]

No thermal recombination and generation implies $J_n$ and $J_p$ are constant throughout the depletion region. Thus, the total current can be define in terms of only the current at the depletion region edges.

\[
J = J_n (-x_p) + J_p (x_n)
\]
Quantitative p-n Diode Solution

Approach:

- Solve minority carrier diffusion equation in quasi-neutral regions
- Determine minority carrier currents from continuity equation
- Evaluate currents at the depletion region edges
- Add these together and multiply by area to determine the total current through the device.
- Use translated axes, $x \rightarrow x'$ and $-x \rightarrow x''$ in our solution.
### Quantitative p-n Diode Solution

**p-type Depletion Region**
- $E = 0$

**n-type**
- $E = 0$

- $x' = 0$
- $x'' = 0$
- $\infty \ x'' = 0$
- $\infty$

\[
0 = D_p \frac{\partial^2 (\Delta p_n)}{\partial x'^2} - \frac{(\Delta p_n)}{\tau_p}
\]

\[
\Delta p_n (x') = A e^{-x'/L_p} + B e^{+x'/L_p}
\]

where $L_p \equiv \sqrt{D_p \tau_p}$

**Boundary Conditions**:

\[
\Delta p_n (x' \to \infty) = 0
\]

\[
\Delta p_n (x' = 0) = \frac{n_i^2}{N_D} \left( e^{\frac{qV_A}{kT}} - 1 \right)
\]


- $B = 0$ and $A = \Delta p_n (x' = 0) = \frac{n_i^2}{N_D} \left( e^{\frac{qV_A}{kT}} - 1 \right)$

\[
\Delta p_n (x') = \frac{n_i^2}{N_D} \left( e^{\frac{qV_A}{kT}} - 1 \right) e^{-x'/L_p} e^{(-x'/L_p)} \text{ for } x' \geq 0
\]
Quantitative p-n Diode Solution

\[
\Delta p_n(x') = \frac{n_i^2}{N_D} \left( e^{\frac{qV_A}{kT}} - 1 \right) e^{(-x'/L_p)} \quad \text{for } x' \geq 0
\]

\[
J_p = -qD_p \frac{d\Delta p_n}{dx}
\]

\[
J_p = q \frac{D_p n_i^2}{L_p N_D} \left( e^{\frac{qV_A}{kT}} - 1 \right) e^{(-x'/L_p)} \quad \text{for } x' \geq 0
\]
Quantitative p-n Diode Solution

p-type

\[ E = 0 \]

Depletion Region

\[ E \neq 0 \]

n-type

\[ E = 0 \]

Similarly for electrons on the p-side...

\[ \Delta n_p (x'') = \frac{n_i^2}{N_A} \left( e^{\frac{qV_A}{kT}} - 1 \right) e^{-x''/L_n} \quad \text{for } x'' \geq 0 \]

\[ J_n = -qD_n \frac{d\Delta n_p}{dx} \]

\[ J_n = qD_n n_i^2 \frac{n_i^2}{L_n N_A} \left( e^{\frac{qV_A}{kT}} - 1 \right) e^{-x''/L_n} \quad \text{for } x'' \geq 0 \]
Quantitative p-n Diode Solution

p-type $E = 0$

Depletion Region $E \neq 0$

n-type $E = 0$

Total current is constant throughout the device. Thus, we can characterize the current flow components as...

$$J = J_n + J_p$$

$p_L x' \propto e^{(-x'/L_p)}$

$n_L x'' \propto e^{(-x''/L_n)}$

Recombination

Recombination
Quantitative p-n Diode Solution

Thus, evaluating the current components at the depletion region edges, we have...

\[ J = J_n(x'' = 0) + J_p(x' = 0) = J_n(x'' = 0) + J_p(x'' = 0) = J_n(x' = 0) + J_p(x' = 0) \]

\[ J = q \left( \frac{D_n n_i^2}{L_n N_A} + \frac{D_p n_i^2}{L_p N_D} \right) \left( e^{qV_a/kT} - 1 \right) \text{ for all } x \]

or

\[ I = I_o \left( e^{qV_a/kT} - 1 \right) \text{ where } I_o = qA \left( \frac{D_n n_i^2}{L_n N_A} + \frac{D_p n_i^2}{L_p N_D} \right) \]

\[ I_o \text{ is the } \text{"reverse saturation current"} \]

Note: Vref from our previous qualitative analysis equation is the thermal voltage, kT/q
Quantitative p-n Diode Solution

Examples: Diode in a circuit

\[ V = 9V, 5V, 2V, -9V \]

\[ 9V = I(1000) + V_A \]

\[ I = I_o \left( e^{\frac{qV_A}{kT}} - 1 \right) \]

where \( I_o = 1 \text{ pA} \)

**Solutions**

<table>
<thead>
<tr>
<th>V</th>
<th>( V_A )</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>9V</td>
<td>0.59V</td>
<td>8.4 mA</td>
</tr>
<tr>
<td>5V</td>
<td>0.58V</td>
<td>4.4 mA</td>
</tr>
<tr>
<td>2V</td>
<td>0.55V</td>
<td>1.5 mA</td>
</tr>
<tr>
<td>-9V</td>
<td>-9.0V</td>
<td>-1 pA</td>
</tr>
</tbody>
</table>

In forward bias (\( V_A > 0 \)) the \( V_A \) is \~\text{constant} for large differences in current

In reverse bias (\( V_A < 0 \)) the current is \~\text{constant} (=saturation current)