ECE 3040
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Homework 2

Unless otherwise specified, assume room temperature (T = 300K) and use the material parameters found in Chapter 2 of Pierret.

1) Purpose: Understanding what the Fermi distribution is telling us.
   Consider an energy state in a semiconductor that is 0.2 eV above the Fermi energy level ($E - E_F = 0.2$ eV).
   a) At T = 0K, what is the probability that the energy state is occupied?
   b) At T = 300K (room temperature), what is the probability that the energy state is occupied?
   c) At T = 500K, what is the probability that the energy state is occupied?
   d) What is the trend observed in parts (a)-(c)?

Ans: (a)We know the Fermi function is,

$$f(E) = \frac{1}{1 + e^{\frac{E - E_F}{kT}}}$$

For $E > E_F$,

$$T \to 0K$$
$$e^{\frac{E-E_F}{kT}} \to \infty$$
$$f(E) \to 0$$

(b) T = 300K,

$$f(E_F + 0.2eV) = \frac{1}{1 + e^{\frac{0.2eV}{0.026}}}$$
$$f(E_F + 0.2eV) = 4.43E-4$$

(c) T = 500K,

$$f(E_F + 0.2eV) = \frac{1}{1 + e^{\frac{0.2eV}{(8.61E-5\frac{eV}{K})(500K)}}}$$

$$f(E_F + 0.2eV) = 0.00951$$

(d) The probability of a state being occupied increases as temperatures increase.

2) Purpose: Understanding the electron distribution in the conduction band.
   Consider a semiconductor with a Fermi level that lays 4kT below the conduction band. Assume the sample is held at room temperature. Which of the following energy levels holds more free electrons?
   A) $E_1 = E_C + 1/2kT$
   B) $E_2 = E_C + 10kT$
   Explain.

Ans: (a) Density of states in conduction band,

$$g_c(E) = \frac{m_n{}^* \sqrt{2m_n{}^*(E-E_C)}}{\pi^2\hbar^3}$$

Fermi function,

$$f(E) = \frac{1}{1+e^{\frac{E-E_F}{kT}}}$$

Electron concentration at energy Energy E,

$$N(E) = g_c(E)f(E) = \frac{m_n{}^* \sqrt{2m_n{}^*(E-E_C)}}{\pi^2\hbar^3} \frac{1}{1+e^{\frac{E-E_F}{kT}}}$$

We can use some simplification as below,

1. Because we are comparing two electron concentrations at different energy levels within the same semiconductor, we can disregard the constant terms in the Density of States expression since they remain consistent in both scenarios. (for ex. $m_n{}^*$ would be same for different energy level)
2. Given Fermi energy is more than 3kT below the conduction band energy, we can use the Boltzmann approximation, $f(E) = \frac{1}{e^{\frac{E-E_F}{kT}}} = e^{\frac{E_F-E}{kT}}$

$$N(E) = g_c(E)f(E) \propto \sqrt{(E-E_C)}e^{\frac{E_F-E}{kT}}$$

(a) $N(E_C + \frac{1}{2}kT) \propto \sqrt{\frac{1}{2}kT}e^{\frac{-4.5kT}{kT}} = \sqrt{(0.5)(0.026eV)}e^{-4.5} = 1.26E-3eV^{\frac{1}{2}}$

(b) $N(E_C + 10kT) \propto \sqrt{10kT}e^{\frac{-14kT}{kT}} = \sqrt{(10)(0.026eV)}e^{-14} = 4.24E-7eV^{\frac{1}{2}}$

Energy 1 will contain more free electrons than Energy 2

3) Purpose: Understanding special cases of doping.
   Concentration questions with a twist.
   a) At room temperature, the electron concentration in a piece of silicon is 10^{16} cm^{-3}. What is the hole concentration?
   b) For a silicon sample maintained at room temperature, the Fermi level is 5kT below the intrinsic Fermi level. What are the carrier concentrations?
   c) A silicon wafer is doped with $N_A = 10^{16}$ cm^{-3} and $N_D = 10^{17}$ cm^{-3}. At T = 0K, what are the equilibrium electron and hole concentrations?

d) At elevated temperatures, a silicon wafer has carrier concentrations n = p = 10^{18} cm^{-3}. What would be the dopant concentration?
e) Knowing that the bandgap of Ge is 0.67 eV. The effective masses of electrons and holes in Ge are $0.055m_0$ and $0.37m_0$. Calculate the intrinsic carrier concentration of Ge.

<u>Hint</u>: Be careful with units for energy calculations.

Ans: (a) Given $n = 10^{16}$ cm^{-3}, we know according to law of mass action,

$$np = n_i^2$$

The intrinsic concentration of silicon is 10^{10} cm^{-3}

$$p = \frac{n_i^2}{n}$$

$$p = \frac{(10^{10} cm^{-3})^2}{10^{16} cm^{-3}} = \frac{10^{20} cm^{-3}}{10^{16} cm^{-3}} = 10^4 cm^{-3}$$

(b) We can use Equations 2.19a and 2.19b to directly solve for n and p.

$$n = n_i e \frac{(E_F - E_i)}{kT}$$

$$p = n_i e \frac{(E_i - E_F)}{kT}$$

$$n = (10^{10} cm^{-3}) e \frac{-5kT}{kT} = 6.73 \times 10^7 cm^{-3} \quad , \quad p = \frac{(10^{10} cm^{-3})^2}{6.73 \times 10^7 cm^{-3}} = 1.48 \times 10^{12} cm^{-3}$$

(c) Regardless of doping concentrations, at T = 0K, no donors or acceptors are ionized in the semiconductor. Therefore $n = p = 0 cm^{-3}$

(d) We can't say anything about dopant concentration. Depending on the temperature level, the semiconductor may enter the "intrinsic temperature" range. In this range, significant thermal energy is introduced to the system, causing bound electrons within the valence band to be elevated across the entire bandgap, transforming them into free electrons within the conduction band. This process results in the creation of holes within the valence band, which can dominate any doping-related effects.

(e) Strategy: Determine Density of States in the conduction band and valence band (N_C and N_V) and use these values and the known bandgap to determine the intrinsic concentration.

Given,

$$m_n^* = 0.055 m_0$$
$$m_p^* = 0.37 m_0$$

We know that,

$$N_C = 2\left[\frac{m_n^* kT}{2\pi\hbar^2}\right]^{\frac{3}{2}} = (2.51 \times 10^{19} cm^{-3})(\frac{m_n^*}{m_0})^{\frac{3}{2}}$$

$$N_C = (2.51\times10^{19} cm^{-3})(0.055)^{\frac{3}{2}} = 1.04\times10^{19} cm^{-3}$$

$$N_V = 2\left[\frac{m_p^* kT}{2\pi\hbar^2}\right]^{\frac{3}{2}} = (2.51\times10^{19} cm^{-3})(\frac{m_p^*}{m_0})^{\frac{3}{2}}$$

$$N_V = (2.51\times10^{19} cm^{-3})(0.37)^{\frac{3}{2}} = 6\times10^{18} cm^{-3}$$

Intrinsic carrier concentration,

$$n_i = \sqrt{N_C N_V} e \frac{-E_G}{2kT}$$

$$n_i = \sqrt{(1.04\times10^{19} cm^{-3})(6\times10^{18} cm^{-3})} e \frac{-0.67eV}{0.052eV}$$

$$n_i = 2.00\times10^{13} cm^{-3}$$

4) Purpose: Understanding the energy band diagram.
Draw the energy band diagram of a hypothetical semiconductor with the following properties:
- T = 300K
- $m_n^* = 0.5m_0$
- $m_p^* = 0.4m_0$
- E_G = 1.1 eV
- n = 5 x 10^{13} cm^{-3}

On your diagram, label E_G, E_C, E_V, E_F and E_i and note their values.
Hint: Do not assume that E_i is at mid-gap.

Ans: Strategy: Determine E_i with the given effective masses. Then determine Density of States in the conduction and valence bands, N_V and N_C, using the effective masses. Use these values to determine the intrinsic concentration of the semiconductor. Use the intrinsic concentration of the semiconductor and the given electron concentration to determine E_F.Draw all relevant energy levels on a diagram, using the valence band energy as a reference (E_V = 0 eV).

$$E_i = \frac{E_C + E_V}{2} + \frac{3}{4} kT \ln(\frac{m_p^*}{m_n^*}) = \frac{E_G}{2} + \frac{3}{4} kT \ln(\frac{m_p^*}{m_n^*})$$

After putting the given values, we can get E_i = 0.545 ev

$$N_C = 2\left[\frac{m_n^* kT}{2\pi\hbar^2}\right]^{\frac{3}{2}} = (2.51\times10^{19} cm^{-3})(\frac{m_n^*}{m_0})^{\frac{3}{2}}$$

$$N_C = 8.87\times10^{18} cm^{-3}$$

We know, $$N_V = 2\left[\frac{m_p^* kT}{2\pi\hbar^2}\right]^{\frac{3}{2}} = (2.51\times10^{19} cm^{-3})(\frac{m_p^*}{m_0})^{\frac{3}{2}}$$

$$N_V = 6.35\times10^{18} cm^{-3}$$

Now putting the values to calculate,

$$n_i = \sqrt{N_C N_V} e \frac{-E_G}{2kT}$$

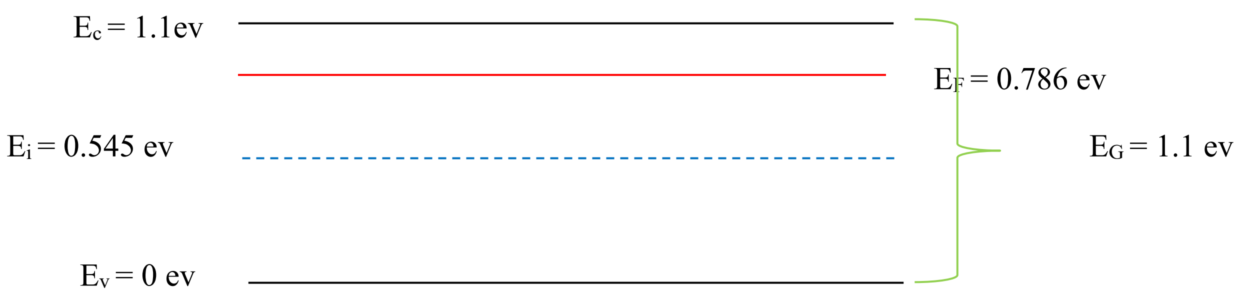
$$n_i = 4.5 \times 10^{19} cm^{-3}$$

Now using the below equations and putting n = 5 x 10^{13} cm^{-3},

$$n = n_i e \frac{(E_F - E_i)}{kT}$$

$$E_F = E_i + kT \ln(\frac{n}{n_i})$$

$$E_F = 0.786eV$$



5) <u>Purpose</u>: Understanding of electron-hole relationships.
   Consider a piece of silicon held at room temperature. For the various doping conditions, find the hole and electron concentrations, the Fermi level, and note whether the sample is n-type or p-type. Assume total ionization of dopants.
   a) 10^{16} cm^{-3} P
   b) 10^{16} cm^{-3} P and 10^{17} cm^{-3} B
   c) 10^{12} cm^{-3} As and 9x10^{11} cm^{-3} B
   <u>Hint</u>: Sections 2.5.5 and 2.5.6 in Pierret may be helpful.

Ans:(a) P (phosphorus) is a donor in silicon. Therefore, we can write,

$$N_D = 10^{16} cm^{-3}$$

$$N_A = 0 cm^{-3}$$

$$N_D - N_A = N_D >> n_i$$

$$n \cong N_D$$

$$n = 10^{16} cm^{-3}$$

$$p = \frac{n_i^2}{N_D} = \frac{(10^{10} cm^{-3})^2}{10^{16} cm^{-3}} = 10^4 cm^{-3}$$

Assuming E_i is at mid gap,

$$E_F = E_i + kT \ln(\frac{n}{n_i})$$

$$E_F = 0.56eV + (0.026eV) \ln(\frac{10^{16} cm^{-3}}{10^{10} cm^{-3}})$$

$$E_F = 0.919eV$$

Therefore the sample is n type.
(b)

$$N_D = 10^{16} cm^{-3}$$
$$N_A = 10^{17} cm^{-3}$$
$$N_A - N_D = N_A >> n_i$$
$$p \cong N_A = 10^{17} cm^{-3}$$
$$n = \frac{n_i^2}{N_A} = \frac{(10^{10} cm^{-3})^2}{10^{17} cm^{-3}} = 10^3 cm^{-3}$$

Putting the values, we get,

$$E_F = E_i - kT \ln(\frac{p}{n_i})$$
$$E_F = 0.143 eV$$

The sample is p type

(c) As (arsenic) is a donor in silicon and B is an acceptor. Therefore,

$$N_A = 9 \times 10^{11} cm^{-3}$$
$$N_D = 10^{12} cm^{-3}$$

This is a case of a "compensated" semiconductor, where N_D and N_A are comparable and nonzero. No simplifications can be made, and we need to carry both terms in our calculations.

$$n = \frac{N_D - N_A}{2} + \left[\left(\frac{N_D - N_A}{2}\right)^2 + n_i^2\right]^{1/2}$$

$$n = \frac{10^{12}\ cm^{-3} - 9\text{x}10^{11}\ cm^{-3}}{2} + \left[\left(\frac{10^{12}\ cm^{-3} - 9\text{x}10^{11}\ cm^{-3}}{2}\right)^2 + (10^{10}\ cm^{-3})^2\right]^{1/2}$$

$$= \frac{10^{11}\ cm^{-3}}{2} + [2.5\text{x}10^{21}\ cm^{-6} + 10^{20}\ cm^{-6}]^{1/2}$$
$$= 5\text{x}10^{10}\ cm^{-3} + 5.10\text{x}10^{10}\ cm^{-3}$$
$$n = 1.01\text{x}10^{11}\ cm^{-3}$$

$$p = \frac{n_i^2}{n} = \frac{(10^{10}\ cm^{-3})^2}{1.01\text{x}10^{11}\ cm^{-3}}$$
$$p = 9.90\text{x}10^{8}\ cm^{-3}$$

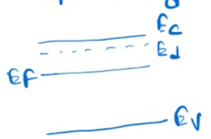
$$E_F = E_i + kT \ln\left(\frac{n}{n_i}\right) = \frac{E_G}{2} + kT \ln\left(\frac{n}{n_i}\right)$$
$$E_F = \frac{1.12\ eV}{2} + (0.0259\ eV) \ln\left(\frac{1.01\text{x}10^{11} cm^{-3}}{10^{10}\ cm^{-3}}\right)$$
$$E_F = 0.620\ eV$$

Sample is n-type.

6) <u>Purpose</u>: Understanding partial ionization.
Find the electron and hole concentrations as well as the Fermi level position for a silicon sample with the following conditions.
a) If a Si sample is doped with $10^{17}cm^{-3}$ P impurities, find the ionized donor density T =300K. For P in Si E_C-E_{donor}=0.045eV.
b) Why is P a more commonly used donor than As?

(a)

$N_d = 10^{17}/cm^3$, we need to find N_d^+



we know,

$$N_d^+ = \frac{N_d}{1+e^{(E_f - E_D)/KT}}$$

$$E_f - E_D = -(E_D - E_F) = -[(E_c - E_f) - (E_c - E_d)]$$

$$N_d^+ = \frac{N_d}{1 + exp - \left[\frac{KT}{q} \ln\left(\frac{N_c}{N_d^+}\right) - 0.045\right]} \quad -(1)$$

$$E_c - E_f = \frac{KT}{q} \ln\left(\frac{N_c}{N_d^+}\right)$$

we will solve the above eqn w/ by iterative method, for Si @ 300K, $N_c = 2.9\times10^{19}/cm^3$

let's say $N_d^+ = 10^{16}/cm^3$, $E_c - E_d = 0.045$

$$= \frac{10^{17}}{1 + exP - \left[0.026 \ln\left(\frac{2.9\times10^{19}}{10^{16}}\right) - 0.045\right]}$$

⋮ after some iteration
⋮

$N_d^+ = 5.3\times10^{16}$ # check for different initial guess of N_d^+

(b) The binding energy of P is situated nearer to the conduction band compared to the binding energy of As, making it a more effective donor