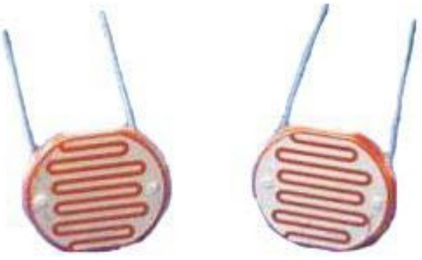
ECE 3040
Dr. Doolittle

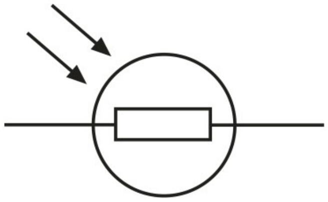
Homework 3

Unless otherwise specified, assume room temperature (T = 300K).

The goal of this homework is to design a light-dependent resistor (LDR), also known as a photoresistor or photocell. An LDR is simply a segment of semiconductor material used as a resistor with a resistance that is a function of illumination. CdS and CdSe are the most common semiconductors used for this application, due to bandgaps that are well-matched to visible and near-infrared wavelengths, respectively. The semiconductor is typically arranged in a long, winding pattern, but for this assignment, simply design the LDR to be one rectangular block.



Two light-dependent resistors



The circuit schematic of a light-dependent resistor

<u>Given</u>:
The LDR is made from CdS, designed to respond to visible light.
The bandgap of CdS is 2.42 eV.
The intrinsic carrier concentration of CdS is 10^3 cm^{-3}.
The semiconductor is n-type with an electron concentration of $5*10^{14}$ cm^{-3}
The electron mobility is 300 cm^2/V-s, and the hole mobility is 10 cm^2/V-s.

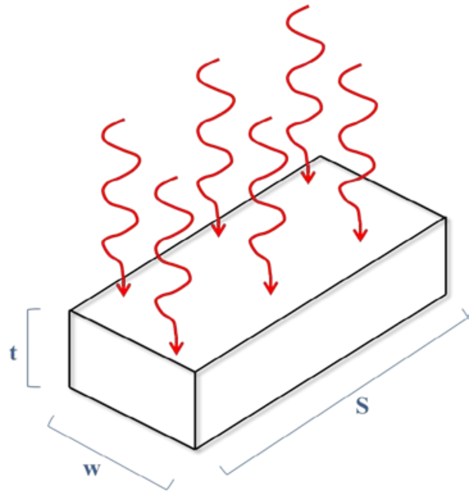
For this assignment it is okay to assume low-level injection for all parts, thus it is valid to use the following formula (Equation 3.34a in Section 3.3.3 of Pierret):

$$\left.\frac{\partial p}{\partial t}\right|_{\substack{\text{i-thermal}\\ \text{R-G}}} = -\frac{\Delta p}{\tau_p}$$

Although, in reality, we would be encountering high-level injection to achieve the amount of variation in resistance we are aiming for, and the use of the more general form would be necessary (Eq. 3.35):

$$\left.\frac{\partial p}{\partial t}\right|_{\substack{\text{i-thermal}\\ \text{R-G}}} = \left.\frac{\partial n}{\partial t}\right|_{\substack{\text{i-thermal}\\ \text{R-G}}} = \frac{n_i^2 - np}{\tau_p(n + n_1) + \tau_n(p + p_1)}$$

1) Purpose: Understanding the relationships between resistance, resistivity, and geometric dimensions of a semiconductor.
It is determined that the CdS film has a width, *w*, of 0.2 mm and a thickness, *t*, of 4 µm. What is the needed length, *S*, to achieve 50 MΩ resistance in the dark? See figure below for reference.
Note: Assume ohmic contacts are applied to both small faces of the bar (the *t* x *w* faces), so that carriers will flow along length *S*.



Ans:

1) we know that the resistance of a material,

$$R = \frac{\rho l}{A}$$

ρ = resistivity of the material
l = length of the block
A = cross sectional area of the plane through which carriers are flowing

we can write, $l = s$ and $A = w \times t$

Therefore,
$$R = \frac{\rho l}{wt}$$

we know the resistivity of a material is defined as (section 3.1.4 in Pierret)

$$\rho = \frac{1}{q(\mu_n n + \mu_p p)}$$

for n type sample of CdS, we can re-write the eqn as,

$$\rho = \frac{1}{q \mu_n N_D}$$

Considering electrons contentration (N_D) will be much higher than the hole concentration,

$$\therefore R = \frac{1}{q \mu_n N_D} \times \frac{s}{wt}$$

$$\therefore s = R q \mu_n N_D w t$$

$$\therefore s = (50 \times 10^6 \Omega) \times (1.6 \times 10^{-19} C) \times (300\ cm^2/V\text{-}s) \times (5 \times 10^{14} cm^{-3}) \times (2 \times 10^{-2} cm) \times (4 \times 10^{-4} cm)$$

$$\approx 9.6\ cm$$

2) Purpose: Understanding generation of electron-hole pairs due to light.
Assume that the only recombination/generation mechanism possible in the semiconductor bar is band-to-band. In addition, assume there is uniform absorption and generation throughout the entire thickness of the film, and the absorption efficiency of the bar is 100% (all eligible incident photons are converted into electron-hole pairs in the semiconductor).

a. If the intensity of the incident light is 2 μW/cm^2 and the wavelengths of the incident photons are 600 nm, calculate the generation rate of electron-hole pairs in the semiconductor, G_L.
b. If the intensity of the incident light is .5 μW/cm^2 and the wavelengths of the incident photons are 512 nm, calculate the generation rate of electron-hole pairs in the semiconductor, G_L.
c. If the intensity of the incident light is 2.5 W/cm^2 and the wavelengths of the incident photons are 512 nm, calculate the generation rate of electron-hole pairs in the semiconductor, G_L.

Hint: The photon flux (number of photons per unit area per unit time) of the incident light can be obtained from the intensity of the light (total energy per unit area per unit time) and the energy of the individual photons. The generation rate is defined as the number of electron-hole pairs generated per unit volume per unit time, which is directly related to the photon flux. Be careful with unit conversions.

Ans:

2)(a) First calculate the energy of one photon having wavelength 600 nm

$$E_{ph} = \frac{hc}{\lambda} = \frac{1.24\ eV.\mu m}{.600\ \mu m} = 2.07\ eV$$

h = planck's constant, c is the speed of light,
λ = photon's wavelength

In this example, band to band generation is the only recombination mechanism possible in the semiconductor bar. Since to generate e-H pairs, the energy of photon must be as large as the bandgap of the semiconductor,

$$E_{ph} < E_g$$

$$\therefore \boxed{G_L = 0}$$

[this is independent of the intensity of the incident light]

(b) $\lambda = 512\ nm = 0.512\ \mu m$

$\therefore$ energy of one photon,

$$E_{ph} = \frac{hc}{\lambda} = \frac{1.24}{0.512} \approx 2.421\ eV$$

$$\therefore E_{ph} \simeq E_g \underline{(2.42\ eV)}$$

because the energy of an incident photon is almost same as bandgap of CdS, a photon will create an e-H pair.

Therefore we can calculate the photon flux as,

$$\varphi_{ph} = \frac{I_0}{E_{ph}} = \frac{.5 \times 10^{-6}\ W/cm^2}{2.42\ (eV) \times 1.6 \times 10^{-19}\ J/eV}$$

$$\varphi_{ph} = 1.29 \times 10^{12}\ cm^{-2} s^{-1}$$

The photon flux is the quantity of incident photons per unit area, with the area being the top surface of the semiconductor bar where the photons impact. As photons are evenly absorbed across the bar's volume, dividing the photon flux by the bar's thickness gives the rate of e-H pair generation caused by incident light,

$$G_L = \frac{\varphi_{ph}}{t} = \frac{1.29\times10^{12}\ cm^{-2}s^{-1}}{4\times10^{-4}\ cm}$$

$$G_L = 3.2\times10^{15}\ cm^{-3}s^{-1}$$

(c) $\lambda = 512\ nm = 0.512\ \mu m$

$E_{ph} = 2.421\ eV$,

$$\therefore \varphi_{ph} = \frac{2.5}{2.421\times1.6\times10^{-19}} = 6.4\times10^{18}\ cm^{-2}s^{-1}$$

$$\therefore G_L = \frac{\varphi_{ph}}{t} = \frac{6.4\times10^{18}}{4\times10^{-4}}$$

$$G_L = 1.6\times10^{22}\ cm^{-3}s^{-1}$$

3) Purpose: Getting familiar with various levels of carrier injection.
If the minority carrier lifetime in the semiconductor is $2\text{x}10^{-6}$ s, what is the resistance of the semiconductor when it is illuminated with the following intensities? Assume the incident photons have a wavelength of 512 nm

a. Intensity is 0.5 μW/cm^2

b. Intensity is 2.5 W/cm^2.

Hint: Your answers for questions 2(b) and 2(c) should come in very handy here.

Ans:

(3) a) we can recall from Q.2(b) that,

$g_L = 3.2 \times 10^{15}\ cm^{-3}.s^{-1}$

As we are assuming steady state condition & the semiconductor is having low-level injection!

$\Delta p = \Delta n = g_L \tau_p = 3.2 \times 10^{15} \times 2 \times 10^{-6}$

$= 6.4 \times 10^{9}\ cm^{-3}$

we know,

$$\rho = \frac{1}{q(\mu_n n + \mu_p p)}$$

The carrier concentrations are,

$n = n_0 + \Delta n = N_D + \Delta n$

$n = 5 \times 10^{14} + 6.4 \times 10^{9}\ cm^{-3}$

$= 5.00 \times 10^{14} /cm^3 \simeq N_D$

$p = p_0 + \Delta p = \frac{n_i^2}{N_D} + \Delta p$

$p = \frac{(10^3)^2}{5 \times 10^{14}} + 6.4 \times 10^9 /cm^3$

$= 6.4 \times 10^9 \approx \Delta p$

Therefore resistivity would be,

$$\rho = \frac{1}{q(\mu_n N_D + \mu_p \Delta p)}$$

$$= \frac{1}{(1.6 \times 10^{-19} \times 300 \times 5 \times 10^{14}) + (1.6 \times 10^{-19} \times 10 \times 6.4 \times 10^{9})}$$

$= 41.67\ \Omega.cm$

$$R = \frac{\rho S}{wt}$$

$$= \frac{41.67 \times 9.6}{2\times10^{-2} \times 4\times10^{-4}}$$

$$\simeq 50\ M\Omega$$

Virtually, there is no change in resistance due to low injection of minority carriers.

(b) Intensity $2.5\ W/cm^2$

from B 2.(c) $h_L = 1.6 \times 10^{22}\ cm^{-3}s^{-1}$

$$\Delta p = \Delta n = h_T \tau_p = (1.6\times10^{22} \times 2\times10^{-6})/cm^3$$

$$= 3.2\times10^{16}\ cm^{-3}$$

Carrier Concentrations are,

$$n = n_0 + \Delta n = N_D + \Delta n$$

$$\Rightarrow n = 5\times10^{14} + 3.2\times10^{16}/cm^3$$

$$n = 3.25\times10^{16}\ cm^{-3} \simeq \Delta n$$

$$p = p_0 + \Delta p = \frac{n_i^2}{N_D} + \Delta p$$

$$\approx \frac{(10^3)^2}{5\times10^{14}} + 3.2\times10^{16}\ cm^{-3}$$

$$\simeq 3.2\times10^{16} \cong \Delta p$$

Resistivity,

$$\rho = \frac{1}{(1.6\times10^{-19} \times 300 \times 5\times10^{14}) + (1.6\times10^{-19} \times 10 \times 3.2\times10^{16})}$$

$$= 0.0752\ \Omega\ cm$$

resistance ,

$$R = \frac{\rho s}{wt}$$

$$R = \frac{0.075 \times 9.6}{2\times10^{-2}\times4\times10^{-4}}$$

$$\simeq 90\, k\Omega$$

Resistance abruptly changed in the high-level injection scenario.

4) Purpose: Understanding Quasi-Fermi levels.
Assume that the intrinsic Fermi level lies exactly at midgap and the minority carrier lifetime is $2\text{x}10^{-6}$ s. Calculate and sketch the Fermi and/or Quasi-Fermi levels of the CdSe in the following conditions:
a. In the dark.
b. Illuminated with 0.5 μW/cm^2 and photons with wavelength of 512 nm.
c. Illuminated with 2.5 W/cm^2 and photons with wavelength of 512 nm.

Ans:

4)(a)

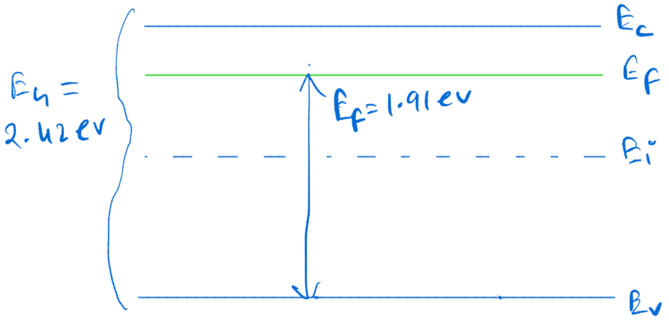
In equilibrium, semiconductor has a fermi level but does not have a quasi fermi level.

using the eq^n from chapter 2 (Eq. 2.19a),

$$n = N_D = n_i e^{(E_f - E_i)/KT}$$

$$E_f = E_i + KT \ln\left(\frac{N_D}{n_i}\right) = \frac{2.42}{2} + (0.026) \times \ln\left(\frac{5\times10^{14}}{10^3}\right)$$

$$E_f = 1.91\, eV$$



(b) Illuminated with $1\,\mu W/cm^2$

$\lambda = 712$ nm

Since the semiconductor is in non-equilibrium, we can consider quasi-fermi level. We will follow the procedure as mentioned in section 3.5.3. Using eqn 3.72a and 3.72b,

$$n \approx N_D = n_i\, e^{(F_N - E_i)/KT}$$

$$F_N \approx E_i + KT \ln\left(\frac{N_D}{n_i}\right) = \frac{2.42}{2} + (0.026\,eV) \times \ln\left(\frac{5\times10^{14}}{10^3}\right)$$

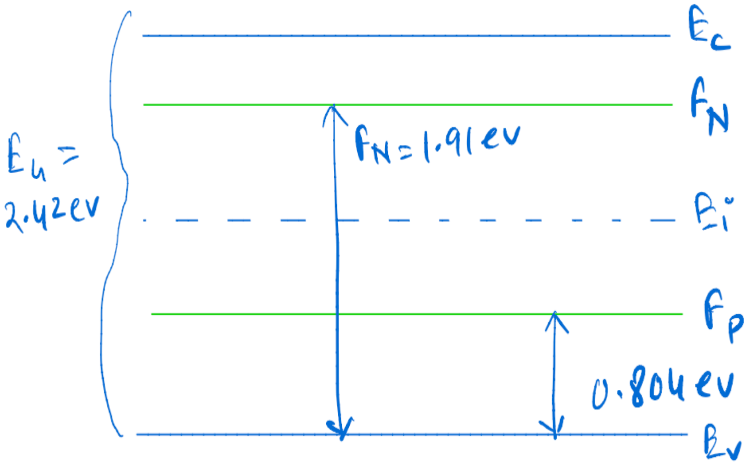
$$F_N = 1.91\ eV$$

Quasi-fermi level is at the same energy as the real fermi level for electrons because the electron concentration level is unchanged in low-level injection.

$$P \approx \Delta P = n_i \, e^{(E_i - F_p)/KT}$$

$$F_p = E_i - KT \ln\left(\frac{\Delta P}{n_i}\right) = \frac{2.42}{2} - (0.0259\ ev) \ln\left(\frac{6.4\times10^{9}}{10^{3}}\right)$$

$$F_p = 0.804\ ev$$



(c) illuminated with 2.5 w/cm² and λ = 512 nm

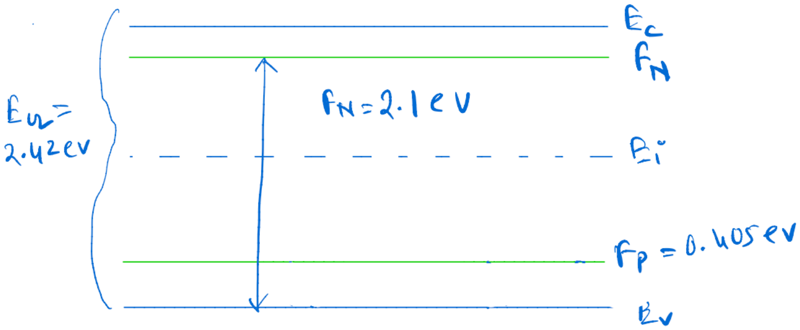
$$n \approx \Delta n \approx n_i \, e^{(F_N - E_i)/KT}$$

$$F_N = E_i + KT \ln\left(\frac{\Delta n}{n_i}\right) = \frac{2.42}{2} + (0.0259\ ev) \ln\left(\frac{3.2\times10^{16}}{10^{3}}\right)$$

$$F_N \approx 2.1\ ev$$

$$P \approx \Delta P = n_i \, e^{(E_i - F_p)/KT}$$

$$F_p = E_i - KT \ln\left(\frac{\Delta P}{n_i}\right) = \underline{0.405\ ev}$$



5) Purpose: Understanding electron and hole drift current.
If 50 V DC is applied across the length of the LDR in various stages of its illumination, what will the electron and hole currents be? Continue to assume the minority carrier lifetime is 2×10^{-6} cm^2/V-s.

a. In the dark.
b. Illuminated with 0.5 µW/cm^2 and photons with wavelength of 512 nm.
c. Illuminated with 2.5 W/cm^2 and photons with wavelength of 512 nm.

Ans:

5)(a) we know total current density in a semiconductor is given by,

$$J = J_N + J_P$$

where J_N = electron current density
and J_P = hole current density

And, $J_N = J_{N,drift} + J_{N,diffusion}$

$$J_P = J_{P,drift} + J_{P,diffusion}$$

In dark condition, since no light is shining on the semiconductor therefore no concentration gradient of carriers (assuming the semiconductor is uniformly doped). Therefore we only have the drift components of current.

$$J_N = J_{N,drift} = q \mu_n n E$$
$$J_P = J_{P,drift} = q p \mu_p E$$

Electric field $(E) = \dfrac{50\,V}{9.6\,cm} = 5.20$ V/cm

at Equilibrium,

$N_D = 5 \times 10^{14}$ /cm^3

$$\therefore P = \frac{n_i^2}{N_D} = \frac{10^6}{5 \times 10^{14}} = \underline{2 \times 10^{-9}}\ cm^{-3}$$

Therefore,

$$J_N = q n \mu_n E$$
$$= 1.6 \times 10^{-19} \times 5 \times 10^{14} \times 300 \times 5.20$$
$$J_N = 0.125\ A/cm^2$$
$$I_N = Wt\, J_N = 2 \times 10^{-2} \times 4 \times 10^{-4} \times 0.125\ A.$$
$$I_N = \underline{1 \times 10^{-6}\ A}$$

Similarly,

$$J_P = q p \mu_p E$$
$$= 1.6 \times 10^{-19} \times 2 \times 10^{-9} \times 10 \times 5.20\ A/cm^2$$
$$= 1.67 \times 10^{-26}\ A/cm^2$$

$$I_P = Wt\, J_P = 2 \times 10^{-2} \times 4 \times 10^{-4} \times 1.67 \times 10^{-26} A$$
$$I_P = \underline{1.33 \times 10^{-31}}\ A$$

current is completely dominated by the electron current, hole current is much smaller.

(b) Illuminated with $0.5\ \mu W/cm^2$, $\lambda = 512\ nm$

Recall low level injection,

$n \approx N_D = 5 \times 10^{14}\ cm^{-3}$

$p \approx \Delta p = 6.4 \times 10^{9}\ cm^{-3}$

$J_N = q \mu_n n \mathcal{E} = 0.125\ A/cm^2$

$\underline{I_N = 1 \times 10^{-6}\ A}$

$J_p = q \mu_p p \mathcal{E}$

$= 1.6 \times 10^{-19} \times 6.4 \times 10^{9} \times 10 \times 5.20\ A/cm^2$

$= 5.3 \times 10^{-8}\ A/cm^2$

$I_p = 2 \times 10^{-2} \times 4 \times 10^{-4} \times 5.3 \times 10^{-8}\ A$

$I_p = \underline{4.26 \times 10^{-13}}\ A$

In this case, the total current is still dominated by electron current. Because the total no. of electrons is hardly affected by the relatively small no. of extra free electrons generated by the light. On the other hand, the hole current experiences a significant surge as the no. of holes witnesses a substantial rise due to the introduction of light-induced carriers. Although hole current remains significantly lower than the electron current by several orders of magnitude.

(c) illuminated with 2.5 W/cm², λ = 512 nm

high level of injection,

$$n \cong \Delta n = 3.25 \times 10^{16}\ cm^{-3}$$

$$p \cong \delta p = 3.2 \times 10^{16}\ cm^{-3}$$

$$J_N = q\mu_n n E = 1.6\times10^{-19} \times 300 \times 3.25\times10^{16} \times 5.20\ A/cm^2$$

$$= 8.11\ A/cm^2$$

$$I_n = 2\times10^{-2} \times 4\times10^{-4} \times 8.11\ A$$

$$= \underline{6.49\times10^{-5}}\ A$$

$$J_p = 1.6\times10^{-19} \times 10 \times 3.2\times10^{16} \times 5.20\ A/cm^2$$

$$\cong 0.27\ A/cm^2$$

$$I_p = \underline{2.13\times10^{-6}}\ A$$

Here, both electron and hole currents undergo a significant increase due to the remarkable rise in electron and hole concentrations. The electron and hole concentrations become nearly equal because the quantity of light-induced carriers far exceeds the intial free electron concentration due to dopants. However the electron current surpasses the hole current by a factor 30×, considered mobility of electron is 30 times greater than the mobility of holes in CdS.

6) Purpose: Understanding minority carrier concentration transients.

Consider the case where the CdSe is illuminated with an intensity of 1 μW/cm² for a very long time, and the light is suddenly turned off at time $t = 0$ s. Sketch and label the hole concentration as a function of time. Denote the hole concentration at time $t = 0$ s, after 3 minority carrier lifetimes have passed, and after 5 minority carrier lifetimes have passed. Assume the photons have a wavelength of 712 nm.

Given:

The LDR is made from CdSe, designed to respond to near-infrared radiation. The bandgap of CdSe is 1.74 eV.

The intrinsic carrier concentration of CdSe is 8000 cm^{-3}.

The semiconductor is n-type with an electron concentration of 10^{13} cm^{-3}.

The electron mobility is 500 cm^2/V-s, and the hole mobility is 10 cm^2/V-s.

CdSe thickness, t is 5 μm and minority carrier lifetime in the semiconductor is $5\text{x}10^{-6}$ s

Ans: As below,

Recall, in the situation of a 1 μW/cm^2 light striking t,

$$\Delta p = G_L\tau_p = (7.16\text{x}10^{15}\ \text{cm}^{-3}\cdot\text{s}^{-1})(5\text{x}10^{-6}\ \text{s}) = 3.58\text{x}10^{10}\ \text{cm}^{-3}$$

From the minority carrier diffusion equation (3.54b), we can say the concentration of excess holes in an n-type semiconductor when a light is turned off takes the following form:

$$\frac{\partial\Delta p}{\partial t} = -\frac{\Delta p}{\tau_p}$$
$$\ln\Delta p = -\frac{t}{\tau_p} + C$$
$$\Delta p(t) = Ce^{-t/\tau_p}$$

Our initial conditions are:

$$\Delta p(0) = C = G_L\tau_p$$
$$\Delta p(\infty) = 0$$

$$\therefore \Delta p(t) = G_L\tau_p e^{-t/\tau_p}$$

At all times,

$$p = p_0 + \Delta p = \frac{n_i^2}{N_D} + \Delta p$$
$$\therefore p(t) = p_0 + \Delta p(t) = p_0 + G_L\tau_p e^{-t/\tau_p}$$

$$\boxed{p(0) = p_0 + \Delta p(0) \cong G_L\tau_p = 3.58\text{x}10^{10}\ \text{cm}^{-3}}$$
$$\boxed{p(3\tau_p) = p_0 + \Delta p(3\tau_p) \cong (3.58\text{x}10^{10}\ \text{cm}^{-3})e^{-3} = 1.78\text{x}10^{9}\ \text{cm}^{-3}}$$
$$\boxed{p(5\tau_p) = p_0 + \Delta p(5\tau_p) \cong (3.58\text{x}10^{10}\ \text{cm}^{-3})e^{-5} = 2.41\text{x}10^{8}\ \text{cm}^{-3}}$$