Dr. Doolittle
Homework 4
Unless otherwise specified, assume room temperature ( $\mathrm{T}=300 \mathrm{~K}$ ).

1) Purpose: Understanding $p-n$ junction band diagrams.

Consider a silicon $p-n$ junction with $N_{\mathrm{A}}=5 \times 10^{14} \mathrm{~cm}^{-3}$ and $N_{\mathrm{D}}=10^{18} \mathrm{~cm}^{-3}$. Draw the band diagram of this device under the following conditions. Solve for and label the Fermi levels on each side, the quasi-Fermi levels, and the potential barrier in each diagram.
a. At equilibrium (applied voltage $V_{\mathrm{A}}=0 \mathrm{~V}$ ).

On the $p$-side,

$$
\begin{gathered}
E_{\mathrm{FP}}=E_{\mathrm{i}}-k T \ln \left(N_{\mathrm{A}} / n_{\mathrm{i}}\right) \\
E_{\mathrm{FP}}=0.56 \mathrm{eV}-(0.0259 \mathrm{eV}) \ln \left(5 \times 10^{14} \mathrm{~cm}^{-3} / 10^{10} \mathrm{~cm}^{-3}\right) \\
E_{\mathrm{FP}}=0.56 \mathrm{eV}-0.280 \mathrm{eV} \\
E_{\mathrm{FP}}=0.280 \mathrm{eV}
\end{gathered}
$$

On the $n$-side,

$$
\begin{gathered}
E_{\mathrm{FN}}=E_{\mathrm{i}}+k T \ln \left(N_{\mathrm{D}} / n_{\mathrm{i}}\right) \\
E_{\mathrm{FN}}=0.56 \mathrm{eV}+(0.0259 \mathrm{eV}) \ln \left(10^{18} \mathrm{~cm}^{-3} / 10^{10} \mathrm{~cm}^{-3}\right) \\
E_{\mathrm{FN}}=0.56 \mathrm{eV}+0.477 \mathrm{eV} \\
E_{\mathrm{FN}}=1.037
\end{gathered}
$$

At equilibrium, the Fermi level is constant throughout the device and the potential barrier is merely the built-in potential.

$$
\begin{gathered}
V_{\mathbf{b i}}=\frac{k T}{q} \ln \left(\frac{N_{\mathrm{A}} N_{\mathrm{D}}}{n_{\mathbf{i}}^{2}}\right) \\
V_{\mathrm{bi}}=(0.0259 \mathrm{~V}) \ln \left(\frac{\left(5 \times 10^{14} \mathrm{~cm}^{-3}\right)\left(10^{18} \mathrm{~cm}^{-3}\right)}{\left(10^{10} \mathrm{~cm}^{-3}\right)^{2}}\right) \\
V_{\mathrm{bi}}=0.757 \mathrm{~V}
\end{gathered}
$$

There are no quasi-Fermi levels in the depletion region.

b. At $V_{\mathrm{A}}=0.5 \mathrm{~V}$.

Regardless of the applied voltage, the Fermi levels on the $\boldsymbol{p}$ - and $\boldsymbol{n}$-side remain unchanged. The potential barrier, however, changes by from $V_{b i}$ to $\left(V_{b i}-V_{\mathrm{A}}\right)$. In addition, there are quasi-Fermi levels that are present in the depletion width and decrease monotonically in the quasi-neutral regions.

c. At $V_{\mathrm{A}}=-1.0 \mathrm{~V}$.

Again, the only change in the band diagram is the change in the potential barrier. However, because the applied bias is negative, the potential barrier increases. The quasi-Fermi levels behave similarly.

2) Purpose: Understanding $p-n$ junction electrostatics.

Consider a silicon $p-n$ junction with $N_{\mathrm{A}}=5 \times 10^{17} \mathrm{~cm}^{-3}$ and $N_{\mathrm{D}}=10^{15} \mathrm{~cm}^{-3}$. The relative permittivity of silicon is 11.8 and the permittivity of free space is $8.85 \times 10^{-14} \mathrm{~F} / \mathrm{cm}$.
a. Determine the magnitude of the depletion width on the $p$-side of the metallurgical junction $\left(x_{\mathrm{p}}\right)$, the depletion width on the $n$-side of the metallurgical junction $\left(x_{\mathrm{n}}\right)$, and the entire depletion width ( $W$ ).

The magnitude of the depletion width depends on the doping concentrations of each side and the material properties of the semiconductor. Referencing Eqs. 5.30 and 5.31 from Pierret,

$$
\begin{aligned}
& V_{\mathrm{bi}}=\frac{k T}{q} \ln \left(\frac{N_{\mathrm{A}} N_{\mathrm{D}}}{n_{\mathrm{i}}^{2}}\right) \\
& V_{\mathbf{b i}}=(0.0259 \mathrm{~V}) \ln \left(\frac{\left(5 \times 10^{17} \mathrm{~cm}^{-3}\right)\left(\mathbf{1 0}^{15} \mathrm{~cm}^{-3}\right)}{\left(10^{10} \mathrm{~cm}^{-3}\right)^{2}}\right) \\
& V_{b i}=0.757 \mathrm{~V} \\
& x_{\mathrm{p}}=\left[\frac{2 \varepsilon_{r} \varepsilon_{0}}{q} \frac{N_{\mathrm{D}}}{N_{\mathrm{A}}\left(N_{\mathrm{A}}+N_{\mathrm{D}}\right)} V_{\mathrm{bi}}\right]^{1 / 2} \\
& =\left[\frac{2(11.8)\left(8.85 \times 10^{-14} \frac{\mathrm{~F}}{\mathrm{~cm}}\right)}{1.6 \times 10^{-19} \mathrm{C}} \frac{10^{15} \mathrm{~cm}^{-3}}{\left(5 \times 10^{17} \mathrm{~cm}^{-3}\right)\left(5.01 \times 10^{17} \mathrm{~cm}^{-3}\right)}(0.757 \mathrm{~V})\right]^{1 / 2} \\
& x_{\mathrm{p}}=1.99 \times 10^{-7} \mathrm{~cm}=1.99 \mathrm{~nm} \\
& x_{\mathrm{n}}=\left[\frac{2 \varepsilon_{r} \varepsilon_{0}}{q} \frac{N_{\mathrm{A}}}{N_{\mathrm{D}}\left(N_{\mathrm{A}}+N_{\mathrm{D}}\right)} V_{\mathrm{bi}}\right]^{1 / 2} \\
& =\left[\frac{2(11.8)\left(8.85 \times 10^{-14} \frac{\mathrm{~F}}{\mathrm{Cm}}\right)}{1.6 \times 10^{-19} \mathrm{C}} \frac{5 \times 10^{17} \mathrm{~cm}^{-3}}{\left(10^{15} \mathrm{~cm}^{-3}\right)\left(5.01 \times 10^{17} \mathrm{~cm}^{-3}\right)}(0.757 \mathrm{~V})\right]^{1 / 2} \\
& x_{\mathrm{n}}=9.93 \times 10^{-5} \mathrm{~cm}=0.993 \mu \mathrm{~m} \\
& W=\left[\frac{2 \varepsilon_{r} \varepsilon_{0}}{q}\left(\frac{N_{\mathrm{A}}+N_{\mathrm{D}}}{N_{\mathrm{A}} N_{\mathrm{D}}}\right) V_{\mathrm{bi}}\right]^{1 / 2} \\
& =\left[\frac{2(11.8)\left(8.85 \times 10^{-14} \frac{\mathrm{~F}}{\mathrm{Cm}}\right)}{1.6 \times 10^{-19} \mathrm{C}} \frac{\left(5.01 \times 10^{17} \mathrm{~cm}^{-3}\right)}{\left(10^{15} \mathrm{~cm}^{-3}\right)\left(5 \times 10^{17} \mathrm{~cm}^{-3}\right)}(0.757 \mathrm{~V})\right]^{1 / 2} \\
& W=9.95 \times 10^{-5} \mathrm{~cm}=0.995 \mu \mathrm{~m}
\end{aligned}
$$

Alternatively, one could solve for either $x_{\mathrm{n}}$ or $x_{\mathrm{p}}$ first and then solve for the other using the relationship $N_{\mathrm{A}} x_{\mathrm{p}}=N_{\mathrm{D}} x_{\mathrm{n}}$. One could also find the total depletion width using the relationship $W=x_{\mathrm{n}}+x_{\mathrm{p}}$. Clearly, there are many ways this question could be answered.
b. Determine the maximum electric field in the depletion width, as well as the electric field at $x=-x_{\mathrm{p}} / 2$.

The maximum electric field in a $p$ - $n$ junction is located directly at the metallurgical junction. After inspecting Eqs. 5.19 and 5.21 in Pierret, one sees that

$$
\begin{gathered}
E_{\max }=E(x=0)=\frac{-q N_{\mathrm{A}}}{\varepsilon_{r} \varepsilon_{0}} x_{\mathrm{p}}=\frac{-q N_{\mathrm{D}}}{\varepsilon_{r} \varepsilon_{0}} x_{\mathrm{n}} \\
E_{\max }=\frac{-\left(1.6 \times 10^{-19} \mathrm{C}\right)\left(5 \times 10^{17} \mathrm{~cm}^{-3}\right)}{(11.8)\left(8.85 \times 10^{-14} \frac{\mathrm{~F}}{\mathrm{~cm}}\right)}\left(1.99 \times 10^{-7} \mathrm{~cm}\right) \\
E_{\max }=-1.52 \times 10^{4} \mathrm{~V} / \mathrm{cm}
\end{gathered}
$$

To evaluate the electric field on the $\boldsymbol{p}$-side of the junction, we must use Equation 5.19

$$
\begin{gathered}
E(x)=\frac{-q N_{\mathrm{A}}}{\varepsilon_{r} \varepsilon_{0}}\left(x_{\mathrm{p}}+x\right) \\
E\left(\frac{-x_{\mathrm{p}}}{2}\right)=\frac{-q N_{\mathrm{A}}}{\varepsilon_{r} \varepsilon_{0}}\left(x_{\mathrm{p}}-\frac{x_{\mathrm{p}}}{2}\right)=\frac{-q N_{\mathrm{A}}}{\varepsilon_{r} \varepsilon_{0}}\left(\frac{x_{\mathrm{p}}}{2}\right) \\
E\left(\frac{-x_{\mathrm{p}}}{2}\right)=\frac{-\left(1.6 \times 10^{-19} \mathrm{C}\right)\left(5 \times 10^{17} \mathrm{~cm}^{-3}\right)}{(11.8)\left(8.85 \times 10^{-14} \frac{\mathrm{~F}}{\mathrm{~cm}}\right)}\left(\frac{1.99 \times 10^{-7} \mathrm{~cm}}{2}\right) \\
E\left(\frac{-x_{\mathrm{p}}}{2}\right)=-7.62 \times 10^{3} \mathrm{~V} / \mathrm{cm}
\end{gathered}
$$

c. Determine the built-in potential $\left(V_{\mathrm{bi}}\right)$, as well as the potential at $x=x_{\mathrm{n}} / 2$.

As solved for already in part (a) to determine $W$, and by referencing Equation 5.10,

$$
V_{\mathrm{bi}}=0.757 \mathrm{~V}
$$

To determine the potential at a given position on the $\boldsymbol{n}$-side of the junction, we must use Equation 5.28

$$
V(x)=V_{b i}-\frac{q N_{\mathrm{D}}}{2 \varepsilon_{r} \varepsilon_{0}}\left(x_{\mathrm{n}}-x\right)^{2}
$$

$$
\begin{gathered}
V\left(\frac{x_{\mathrm{n}}}{2}\right)=V_{\mathrm{bi}}-\frac{q N_{\mathrm{D}}}{2 \varepsilon_{r} \varepsilon_{0}}\left(x_{\mathrm{n}}-\frac{x_{\mathrm{n}}}{2}\right)^{2}=V_{\mathrm{bi}}-\frac{q N_{\mathrm{D}}}{2 \varepsilon_{r} \varepsilon_{0}}\left(\frac{x_{\mathrm{n}}}{2}\right)^{2} \\
V\left(\frac{x_{\mathrm{n}}}{2}\right)=0.757 \mathrm{~V}-\frac{\left(1.6 \times 10^{-19} \mathrm{C}\right)\left(10^{15} \mathrm{~cm}^{-3}\right)}{2(11.8)\left(8.85 \times 10^{-14} \frac{\mathrm{~F}}{\mathrm{~cm}}\right)}\left(\frac{9.93 \times 10^{-5} \mathrm{~cm}}{2}\right)^{2} \\
V\left(\frac{x_{\mathrm{n}}}{2}\right)=0.568 \mathrm{~V}
\end{gathered}
$$

3) Purpose: Understanding junction capacitance.

Consider the $p-n$ junction described in Question 2 above. Assume the cross-sectional area of the diode is $2 \times 10^{-5} \mathrm{~cm}^{2}$.
a. Determine the equilibrium junction capacitance ( $C_{\mathrm{J} 0}$ ).

## Equation 7.2,

$$
C_{\mathrm{J} 0}=\frac{(11.8)\left(8.85 \times 10^{-14} \frac{\varepsilon_{r} \varepsilon_{0} A}{W}\right.}{\mathbf{\mathrm { cm }})\left(2 \times 10^{-5} \mathrm{~cm}^{2}\right)} \mathrm{C}_{\mathrm{J} 0}=2.95 \times 10^{-5} \mathrm{~cm} 10 \times 10^{-13} \mathrm{~F} \quad .
$$

b. The diode is to be used in an LC circuit with a $10-\mathrm{nH}$ inductor. If the desired oscillation frequency of the circuit is $f=5 \mathrm{GHz}$, what should the diode be biased at?

In an LC circuit, the oscillation frequency is related to the capacitance and inductance by the following relationship:

$$
f=\frac{1}{2 \pi \sqrt{L C}}
$$

Therefore, the capacitance required to obtain an oscillation frequency of $5 \mathbf{G H z}$ is

$$
C=\frac{1}{L(2 \pi f)^{2}}=\frac{1}{\left(10^{-8} \mathrm{H}\right)\left(2 \pi\left(5 \times 10^{9} \mathrm{~Hz}\right)\right)^{2}}=1.01 \times 10^{-13} \mathrm{~F}
$$

Equation 7.9 describes the relationship between the equilibrium junction capacitance and the capacitance of the diode when there is an applied bias.

$$
\begin{gathered}
C_{\mathrm{J}}=\frac{C_{\mathrm{J} 0}}{\left(1-V_{\mathrm{A}} / V_{\mathrm{bi}}\right)^{1 / 2}} \\
=(0.757 \mathrm{~V})\left[1-(2.10 / 1.01)^{2}\right] \\
V_{\mathrm{J}}=V_{\mathrm{bi}}\left[1-\left(C_{\mathrm{J}} / C_{\mathrm{J}}\right)^{2}\right] \\
V_{\mathrm{A}}=-2.52 \mathrm{~V}
\end{gathered}
$$

4) Purpose: Understanding $p-n$ junction carrier concentrations.

Consider a silicon $p-n$ junction with $N_{\mathrm{A}}=10^{16} \mathrm{~cm}^{-3}$ and $N_{\mathrm{D}}=10^{18} \mathrm{~cm}^{-3}$. Let the mobility of electrons $\left(\mu_{\mathrm{n}}\right)$ be $1300 \mathrm{~cm}^{2} / \mathrm{V}$-s and the mobility of holes $\left(\mu_{\mathrm{p}}\right)$ be $400 \mathrm{~cm}^{2} / \mathrm{V}$-s. Let the minority carrier lifetime electrons and holes ( $\tau_{\mathrm{n}}$ and $\tau_{\mathrm{p}}$, respectively) both be $10^{-6} \mathrm{~s}$. There is an applied voltage of 0.6 V .
a. Solve for the excess electron concentrations on the $p$-side of the junction at $130 \mu \mathrm{~m}$, $580 \mu \mathrm{~m}$, and $1160 \mu \mathrm{~m}$ away from the depletion width edge (i.e. moving further into the $p$-side of the device).

Equation 6.25, shown below, is the expression for excess electrons on the $p$-side of $\boldsymbol{p}$ - $\boldsymbol{n}$ junction. Before we can use it, however, we must determine what the minority carrier diffusion length of electrons is.

$$
\Delta n_{p}(x)=\frac{n_{\mathrm{i}}^{2}}{N_{\mathrm{A}}}\left(e^{q V_{\mathrm{A}} / k T-1}\right) e^{-x / L_{\mathrm{N}}}
$$

From the Einstein relationship, we find the diffusion coefficient of electrons.

$$
D_{\mathrm{N}}=\mu_{\mathrm{n}} \frac{k T}{q}=\left(1300 \mathrm{~cm}^{2} / \mathrm{Vs}\right)(0.0259 \mathrm{~V})=37.67 \mathrm{~cm}^{2} / \mathrm{s}
$$

From Equation 3.69, we find the minority carrier diffusion length.

$$
L_{\mathrm{N}}=\sqrt{D_{\mathrm{N}} \tau_{n}}=\sqrt{\left(37.67 \mathrm{~cm}^{2} / \mathrm{s}\right)\left(10^{-6} \mathrm{~s}\right)}=5.80 \times 10^{-3} \mathrm{~cm}
$$

Now, solving for excess electrons,

$$
\begin{aligned}
\Delta n_{p}(x)= & \frac{\left(10^{10} \mathrm{~cm}^{-3}\right)^{2}}{10^{16} \mathrm{~cm}^{-3}}\left(e^{0.6 \mathrm{~V} / 0.0259 \mathrm{v}-1) e^{-x / 5.80 \times 10^{-3} \mathrm{~cm}}} \begin{array}{c} 
\\
=\left(1.15 \times 10^{14} \mathrm{~cm}^{-3}\right) e^{-x / 5.80 \times 10^{-3} \mathrm{~cm}} \\
\Delta n_{p}(0.013 \mathrm{~cm})=\left(1.15 \times 10^{14} \mathrm{~cm}^{-3}\right) e^{-2.24} \\
\Delta n_{p}(0.013 \mathrm{~cm})=1.22 \times 10^{13} \mathrm{~cm}^{-3} \\
\Delta n_{p}(0.058 \mathrm{~cm})=\left(1.15 \times 10^{14} \mathrm{~cm}^{-3}\right) e^{-10} \\
\Delta n_{p}(0.058 \mathrm{~cm})=5.22 \times 10^{9} \mathrm{~cm}^{-3} \\
\Delta n_{p}(0.116 \mathrm{~cm})=\left(1.15 \times 10^{14} \mathrm{~cm}^{-3}\right) e^{-20} \\
\Delta n_{p}(0.116 \mathrm{~cm})=2.37 \times 10^{5} \mathrm{~cm}^{-3}
\end{array}\right.
\end{aligned}
$$

b. Solve for the excess hole concentration on the $n$-side of the junction at $70 \mu \mathrm{~m}$ and $1000 \mu \mathrm{~m}$ away from the depletion width edge.

We must follow a similar procedure to that of part (a).

$$
\left.\begin{array}{c}
D_{\mathrm{P}}=\mu_{\mathrm{p}} \frac{k T}{q}=\left(400 \mathrm{~cm}^{2} / \mathrm{Vs}\right)(0.0259 \mathrm{~V})=10.36 \mathrm{~cm}^{2} / \mathrm{s} \\
L_{\mathrm{P}}=\sqrt{D_{\mathrm{P}} \tau_{p}}=\sqrt{\left(10.36 \mathrm{~cm}^{2} / \mathrm{s}\right)\left(10^{-6} \mathrm{~s}\right)}=3.22 \times 10^{-3} \mathrm{~cm} \\
\Delta p_{n}(x)=\frac{n_{\mathrm{i}}^{2}}{N_{\mathrm{D}}}\left(e^{q V_{\mathrm{A}} / k T-1}\right) e^{-x / L_{\mathrm{P}}} \\
\Delta p_{n}(x)=\frac{\left(10^{10} \mathrm{~cm}^{-3}\right)^{2}}{10^{18} \mathrm{~cm}^{-3}}\left(e^{0.6 \mathrm{~V} / 0.0259 \mathrm{~V}-1) e^{-x / 3.22 \times 10^{-3} \mathrm{~cm}}}\right. \\
=\left(1.15 \times 10^{12} \mathrm{~cm}^{-3}\right) e^{-x / 3.22 \times 10^{-3} \mathrm{~cm}} \\
\Delta p_{n}(0.007 \mathrm{~cm})=\left(1.15 \times 10^{12} \mathrm{~cm}^{-3}\right) e^{-2.17} \\
\Delta p_{n}(0.007 \mathrm{~cm})=1.29 \times 10^{11} \mathrm{~cm}^{-3}
\end{array}\right] \begin{gathered}
\Delta p_{n}(0.1 \mathrm{~cm})=\left(1.15 \times 10^{12} \mathrm{~cm}^{-3}\right) e^{-31} \\
\Delta p_{n}(0.1 \mathrm{~cm})=0.03 \mathrm{~cm}^{-3}
\end{gathered}
$$

5) Purpose: Understanding $p-n$ junction $I-V$ characteristics.

Consider the same diode described in Question 3. What is the reverse saturation current $\left(I_{0}\right)$ of the diode? What is the current in the diode when the applied voltage $\left(V_{\mathrm{A}}\right)$ is 0.6 V ?

Referencing Eqs. 6.29 and 6.30,

$$
\begin{gathered}
I_{0}=q A\left(\frac{D_{\mathrm{N}}}{L_{\mathrm{N}}} \frac{n_{\mathrm{i}}^{2}}{N_{\mathrm{A}}}+\frac{D_{\mathrm{P}}}{L_{\mathrm{P}}} \frac{n_{\mathrm{i}}^{2}}{N_{\mathrm{D}}}\right) \\
I_{0}=\left(1.6 \times 10^{-19} \mathrm{C}\right)\left(2 \times 10^{-5} \mathrm{~cm}^{2}\right)\left(\frac{\left(33.67 \mathrm{~cm}^{2} / \mathrm{Vs}\right)}{\left(5.80 \times 10^{-3} \mathrm{~cm}\right)} \frac{\left(10^{10} \mathrm{~cm}^{-3}\right)^{2}}{\left(10^{16} \mathrm{~cm}^{-3}\right)}\right. \\
\left.+\frac{\left(10.36 \mathrm{~cm}^{2} / \mathrm{Vs}\right)}{\left(3.22 \times 10^{-3} \mathrm{~cm}\right)} \frac{\left(10^{10} \mathrm{~cm}^{-3}\right)^{2}}{\left(10^{18} \mathrm{~cm}^{-3}\right)}\right) \\
I_{0}=1.87 \times 10^{-16} \mathrm{~A} \\
I=I_{0}\left(e^{q V_{\mathrm{A}} / k T-1}\right) \\
I=\left(1.87 \times 10^{-16} \mathrm{~A}\right)\left(e^{0.6 \mathrm{~V} / 0.0259 \mathrm{~V}-1}\right) \\
I=2.15 \times 10^{-6} \mathrm{~A}
\end{gathered}
$$

6) Purpose: Understanding how diodes behave in circuits.

Find the Q-points for the three diodes in the circuit below. Use the constant voltage drop model for the diodes, with $V_{o n}=0.7 \mathrm{~V}$.
Hint: See Example 3.8 in Jaeger \& Blalock.


In multi-diode circuits with $N$ number of diodes, there are $2^{N}$ possible modes the circuit could be operating in: each diode can be either ON or OFF. Circuits can quickly be overwhelming to try to solve by hand. The strategy outlined in Jaeger \& Blalock is to start by assuming every diode is ON, solve for the Q-point of each diode (its DC current and voltage), and then check to see if the assumption of the state of each diode is correct. A diode that is $O N$ will have a positive current and, in the constant voltage drop model, a voltage drop of $+\mathbf{0 . 7} \mathrm{V}$. A diode that is OFF will have no current flowing through it and a voltage that is less than +0.7 V . If an assumption is incorrect, you will know because the $Q$-point of the diode will not match the above description. However you go about testing possible solutions for this particular circuit, eventually you will test the assumption that $D_{2}$ and $D_{3}$ are on, and $D_{1}$ is off. This is the correct assumption.


The circuit above is the constant voltage drop (CVD) model of the circuit in question, with $D_{2}$ and $D_{3} O N$ and $D_{1}$ OFF.

To solve for the Q-points, we'll employ Kirchoff's voltage law along two branches of the circuit.

$$
\begin{aligned}
& 10 \mathrm{~V}-(10 \mathrm{k} \Omega) I_{1}-0.7 \mathrm{~V}-(10 \mathrm{k} \Omega) I_{4}=-1.5 \mathrm{~V} \\
& 10 \mathrm{~V}-(2.5 \mathrm{k} \Omega) I_{2}-0.7 \mathrm{~V}-(10 \mathrm{k} \Omega) I_{4}=-1.5 \mathrm{~V}
\end{aligned}
$$

Using Kirchoff's current law, we can figure out that:

$$
\begin{gathered}
I_{2}+I_{3}=I_{4} \\
I_{2}=I_{D 2} \\
I_{3}=I_{D 3}
\end{gathered}
$$

From the above system of equations, we obtain

$$
\begin{aligned}
& I_{D 2}=0.18 \mathrm{~mA} \\
& I_{D 3}=0.72 \mathrm{~mA}
\end{aligned}
$$

And we can determine that

$$
V_{D 1}=-3.2 \mathrm{~V}
$$

Which leaves us with the correct Q-points of

$$
\begin{aligned}
Q_{1} & =(0 \mathrm{~mA},-3.2 \mathrm{~V}) \\
Q_{2} & =(0.18 \mathrm{~mA}, 0.7 \mathrm{~V}) \\
\boldsymbol{Q}_{3} & =(0.72 \mathrm{~mA}, 0.7 \mathrm{~V})
\end{aligned}
$$

7) Purpose: Using diodes to shape signal waveform.

Calculate output voltage, $v_{o}$, for the following circuit. Show the graphical representation of $v_{O}$ for $-20 \mathrm{~V} \leq v_{I} \leq+20 \mathrm{~V}$, assuming ideal diodes. Show all work for full credit.


## For $-20 \mathrm{~V} \leq v_{I}<-6 \mathrm{~V}$

Left diode is conducting and right diode is off.
So, the output voltage is

$$
v_{o}=-6 \mathrm{~V}
$$

For $-6 \mathrm{~V} \leq v_{I}<4 \mathrm{~V}$
Both diodes are off. No current flows during this voltage range.
So, the output voltage is

$$
v_{O}=v_{I}
$$

For $4 \mathrm{~V} \leq v_{I}<20 \mathrm{~V}$
Left diode is off and right diode is conducting. Current through the right branch is

$$
i=\frac{v_{I}+4}{2.5 \mathrm{k}+7.5 \mathrm{k}} \mathrm{~A}
$$

So, the output voltage is

$$
v_{o}=4+i \times 7.5 \mathrm{k}=\frac{3}{4} v_{I}+1 \mathrm{~V}
$$


8) Purpose: Implementing diodes in practical circuits.

A particular design of a voltage regulator circuit is shown in the next page. Diodes $D_{1}$ and $\mathrm{D}_{2}$ each has a voltage drop of 0.65 V at 1.4 mA .
(a) What regulator voltage output, $V_{O}$, when a $1 \mathrm{k} \Omega$ resistance is not connected (no load condition)?
(b) What is $V_{O}$ when a $1 \mathrm{k} \Omega$ resistance is added as load?

Hint: Use diode exponential model and iterative process to solve $V_{O}$.

(a) Both diodes are 0.65 V @ 1.4 mA current. With no load, $I_{L}=0$ and $I=I_{D}$. Using iteration to find $V_{o}$ and $I_{D}$.

| Step 1: | $I_{D}=\frac{5 \mathrm{~V}-2 \times 0.65 \mathrm{~V}}{1.2 \mathrm{k} \Omega}=3.0833 \mathrm{~mA}$ |
| :--- | :---: |
|  | $V_{2}=V_{1}+2.3 V_{T} \log \left(\frac{I_{2}}{I_{1}}\right)$ |
|  | $V_{2}=0.65 \mathrm{~V}+2.3 \times 0.025 \times \log \left(\frac{3.0833}{1.4}\right)=0.6697 \mathrm{~V}$ |
| Step 2: | $I_{D}=\frac{5 \mathrm{~V}-2 \times 0.6697 \mathrm{~V}}{1.2 \mathrm{k} \Omega}=3.0505 \mathrm{~mA}$ |
|  | $V_{2}=0.65 \mathrm{~V}+2.3 \times 0.025 \times \log \left(\frac{3.0833}{1.4}\right)=0.6694 \mathrm{~V}$ |
| Step 3: | $I_{D}=\frac{5 \mathrm{~V}-2 \times 0.6694 \mathrm{~V}}{1.2 \mathrm{k} \Omega}=3.0505 \mathrm{~mA}$ |

Step 3 is similar to step 2, so we stop the iterations here.

$$
\begin{gathered}
V_{D}=0.6694 \mathrm{~V} \& I_{D}=3.051 \mathrm{~mA} \\
V_{O}=2 \times 0.6694 \mathrm{~V}=1.339 \mathrm{~V}
\end{gathered}
$$

(b) A load of $1 \mathrm{k} \Omega$ is now connected. Again, iteration will be used to find $V_{\boldsymbol{O}}$.

| Step 1: | $\begin{gathered} I_{L}=\frac{1.339 \mathrm{~V}}{1 \mathrm{k} \Omega}=1.339 \mathrm{~mA} \\ I_{D}=I-I_{L}=3.05 \mathrm{~mA}-1.339 \mathrm{~mA}=1.712 \mathrm{~mA} \\ V_{o}=2 \times V_{D}=2\left[0.65 \mathrm{~V}+2.3 \times 0.025 \times \log \left(\frac{1.712}{1.4}\right)\right] \\ =1.31005 \mathrm{~V} \end{gathered}$ |
| :---: | :---: |
| Step 2: | $\begin{gathered} I=\frac{5 \mathrm{~V}-1.31005 \mathrm{~V}}{1.2 \mathrm{k} \Omega}=3.075 \mathrm{~mA} \\ I_{L}=\frac{1.31005 \mathrm{~V}}{1 \mathrm{k} \Omega}=1.31005 \mathrm{~mA} \\ I_{D}=3.075 \mathrm{~mA}-1.31005 \mathrm{~mA}=1.765 \mathrm{~mA} \\ V_{O}=2 \times V_{D}= \\ =2\left[0.65 \mathrm{~V}+2.3 \times 0.025 \times \log \left(\frac{1.765}{1.4}\right)\right] \\ =1.3116 \mathrm{~V} \end{gathered}$ |
| Step 3: | $\begin{gathered} I=\frac{5 \mathrm{~V}-1.3116 \mathrm{~V}}{1.2 \mathrm{k} \Omega}=3.074 \mathrm{~mA} \\ I_{L}=\frac{1.3116 \mathrm{~V}}{1 \mathrm{k} \Omega}=1.3116 \mathrm{~mA} \\ I_{D}=3.074 \mathrm{~mA}-1.3116 \mathrm{~mA}=1.762 \mathrm{~mA} \\ V_{o}=2 \times V_{D}=2\left[0.65 \mathrm{~V}+2.3 \times 0.025 \times \log \left(\frac{1.765}{1.4}\right)\right] \\ =1.3115 \mathrm{~V} \end{gathered}$ |
| Step 4: | $\begin{gathered} I=\frac{5 \mathrm{~V}-1.3115 \mathrm{~V}}{1.2 \mathrm{k} \Omega}=3.074 \mathrm{~mA} \\ I_{L}=\frac{1.3115 \mathrm{~V}}{1 \mathrm{k} \Omega}=1.3115 \mathrm{~mA} \\ I_{D}=I-I_{L}=3.074 \mathrm{~mA}-1.3116 \mathrm{~mA}=1.762 \mathrm{~mA} \end{gathered}$ |

Step 4 is similar to step 3, so we stop the iterations here.

$$
V_{o}=1.3115 \mathrm{~V}
$$

9) Purpose: Investigating the full-wave bridge rectifier.

Consider the following full wave rectifier circuit. The supply voltage from wall-point is stepped down using a transformer and connected to the input of the full wave rectifier. Here, the input voltage $\left(v_{I}\right)$ to the rectifier is a sine-wave with amplitude of 10 V . Assume that diodes can be represented by the constant-voltage-drop model with $V_{D}=$ 0.6 V .
(a) Show the current path for the positive and negative half-cycle of $v_{I}$. Also, sketch and clearly label the transfer characteristic ( $v_{O}$ vs. $v_{I}$ ) and the time response (same plot showing $v_{O}$ vs. $t$ and $v_{I}$ vs. $t$ ) of the circuit shown.


For the positive half cycle of $v_{I}$, that is when $v_{I}>2 \times V_{D}=1.2 \mathrm{~V}$,


For the negative half cycle of $v_{I}$, that is when $v_{I}<-2 \times V_{D}=-1.2 \mathrm{~V}$,


The time response of the input and output voltages is


The transfer characteristic of the circuit is

(b) The output resistance, $R$ is now replaced with a Zener diode, $Z$. Assume that the Zener voltage is 6.5 V and that $r_{z}$ is negligibly small. How does the transfer characteristic and time response change? Generate both plots for the modified circuit.


If the resistor is replaced with a Zener diode, it will produce a square wave at the output of the rectifier. The time response and transfer characteristic are given below.

The time response of the input and output voltages is


The transfer characteristic of the circuit is


