Lecture 26

MOSFET Small Signal Model

Reading: Jaeger 13.7 and Notes
MOSFET Small Signal Model and Analysis

• Just as we did with the BJT, we can consider the MOSFET amplifier analysis in two parts:
  • Find the DC operating point
  • Then determine the amplifier output parameters for very small input signals.
MOSFET Small Signal Model and Analysis

Non-Linear I-V relationship (BJT, MOSFET, etc...)

\[ I_1 = y_{11} V_1 + y_{12} V_2 \]
\[ I_2 = y_{21} V_1 + y_{22} V_2 \]

Linearize over “small signal range”

Linear Two Port Network

\[ I_G = y_{11} V_G + y_{12} V_D \]
\[ I_D = y_{21} V_G + y_{22} V_D \]

General “y-parameter” Network

MOSFET “y-parameter” Network
MOSFET Small Signal Model and Analysis

\[
\begin{bmatrix}
I_{GS} \\
I_{DS}
\end{bmatrix} =
\begin{bmatrix}
y_{11} & y_{12} \\
y_{21} & y_{22}
\end{bmatrix}
\begin{bmatrix}
V_{GS} \\
V_{DS}
\end{bmatrix}
\]

\[I_{GS} = y_{11} V_{GS} + y_{12} V_{DS}\]

\[I_{DS} = y_{21} V_{GS} + y_{22} V_{DS}\]

\[y_{ij} = \left. \frac{\partial I_j}{\partial V_i} \right|_{V_{GS},Q,V_{DS},Q}\]

Derivative of current-voltage equation evaluated at the Quiescent Point

MOSFET Amplifiers are biased into Saturation (or Active Mode)

\[I_{DS} = \frac{K^n}{2} \left( (V_{GS} - V_{TN})^2 \right) (1 + \lambda V_{DS}) \quad \text{for} \quad V_{DS} \geq V_{GS} - V_{TN}\]

1.) Input Conductance

\[I_{GS} = 0 \quad \Rightarrow \quad \frac{\partial I_{GS}}{\partial V_{GS}} = 0 \quad \text{and} \quad \frac{\partial I_{GS}}{\partial V_{DS}} = 0 \quad \Rightarrow \quad y_{11} = 0 \quad \text{and} \quad y_{12} = 0\]

2.) Output Conductance

\[\frac{\partial I_{DS}}{\partial V_{DS}} = y_{22} = \frac{\lambda K^n}{2} (V_{GS} - V_T)^2\]

3.) Transconductance

\[\frac{\partial I_{DS}}{\partial V_{GS}} = y_{21} = K_n (V_{GS} - V_T) (1 + \lambda V_{DS})\]
MOSFET Small Signal Model and Analysis

Compare with BJT Results

There is a large amount of symmetry between the MOSFET and the BJT

\[ y_{22} = g_o = \frac{\lambda}{2} K_n (V_{GS} - V_T)^2 = \frac{I_{DS}}{1 + V_{DS}} \]

\[ y_{21} = g_m = K_n (V_{GS} - V_T)(1 + \lambda V_{DS}) = \frac{I_{DS}}{\left( \frac{V_{GS} - V_{TN}}{2} \right)} \]

\[ y_{22} = \frac{I_C}{V_A + V_{CE}} \]

\[ y_{21} = \frac{I_C}{V_T} \]

Each of these parameters act in the same manner.
MOSFET Small Signal Model and Analysis

Putting the mathematical model into a small signal equivalent circuit

Compare this to the BJT small signal equivalent circuit
Calculate the voltage gain, \( A_v = \frac{v_o}{v_s} \)

Given: \( K_n = 1 \text{ mA/V}^2, \quad \lambda = 0.015 \text{ V}^{-1} \)

Bias Point of: \( I_{DS} = 2 \text{ mA}, \quad V_{DS} = 7.5 \text{V} \)
MOSFET Small Signal Model and Analysis

Example: Jaeger 13.94

\[ g_o = \frac{\lambda}{2} K_n (V_{GS} - V_T)^2 \]
\[ g_m = K_n (V_{GS} - V_T)(1 + \lambda V_{DS}) \]

Need to find \( V_{GS} - V_T \)

\[ I_{DS} = \frac{K_n}{2} \left[ (V_{GS} - V_{TN})^2 \right] (1 + \lambda V_{DS}) \]

\[ 2 mA = \frac{1 mA/V^2}{2} \left[ (V_{GS} - V_{TN})^2 \right] (1 + 0.015 (7.5)) \]

\[ 1.9 V = \sqrt{\frac{4}{1.11}} \]

\[ V_{GS} - V_{TN} = 1.9 V \]

\[ g_m = 2.11 mS \quad g_o = 27.1 \mu S \Rightarrow r_o = 36.9 k\Omega \]
MOSFET Small Signal Model and Analysis

Example: Jaeger 13.94

\[
A_v = \frac{v_o}{v_s} = \frac{v_{GS}}{v_s} \frac{v_o}{v_{GS}}
\]

\[
\frac{v_{GS}}{v_s} = \frac{1\text{Meg}}{10k + 1\text{Meg}} = 0.99 \quad \text{and} \quad \frac{v_o}{v_{GS}} = -g_m \left( r_o \left| R_d \right| R_3 \right) = -2.1mS(3.48k) = -7.35
\]

\[
\therefore A_v = \frac{v_o}{v_s} = \frac{v_{GS}}{v_s} \frac{v_o}{v_{GS}} = -7.27 \quad [V/V]
\]
MOSFET Amplifiers

What is the Maximum Gain Possible?

Is it saturated (Constant current), \( |V_{DS}| \geq |V_{GS} - V_{TP}| \geq 0 \)

but \( V_{GS} = 0 \)

and \( V_{TP} \geq 0 \) for a depletion mode MOSFET so,

\( V_{DS} \) (a positive number) > \((-V_{TP} \) a negative number)

⇒ Is Saturated!

\[ A_{v,Max} = -g_m r_o \]

\[ A_{v,Max} = -\frac{K_n (V_{GS} - V_T)(1 + \lambda V_{DS})}{\lambda K_n (V_{GS} - V_T)^2} \]

\[ A_{v,Max} = -\frac{(1 + \lambda V_{DS})}{\lambda (V_{GS} - V_T)} \]

g_o is internal to the transistor and can not be avoided. Any additional resistor due to external circuitry will lower the gain. For this reason current sources are often used as the “load” instead of bias resistors in amplifier circuits.
MOSFET Small Signal Model and Analysis

Add in capacitances

Overlap of Gate Oxide

Gate to channel to Bulk capacitance

Reverse Bias Junction capacitances

Overlap of Gate Oxide

$V_d > V_g - V_t$
Complete Model of a MOSFET

Due to effective modulation of the threshold voltage.

\[ g_{mb} = g_m \frac{\gamma}{2\sqrt{V_{SB}} + 2\phi_F} \]

Overlap of Gate Oxide

Overlap of Gate Oxide and source

Gate to channel to Bulk capacitance

Reverse Bias Junction capacitances
MOSFET Small Signal Model and Analysis

SPICE MOSFET Model
SPICE models the drain current ($I_{DS}$) of an n-channel MOSFET using the following parameters/equations (SPICE variables are shown in ALL CAPITAL LETTERS)

Cutoff:  
$$I_{DS} = 0$$

Linear:
$$I_{DS} = \frac{KP}{2} \left( \frac{W}{L_{EFF}} \right) V_{DS} \left[ 2(V_{GS} - V_{TH}) - V_{DS} \right] \left( 1 + (\lambda \cdot V_{DS}) \right)$$

Saturation:
$$I_{DS} = \frac{KP}{2} \left( \frac{W}{L_{EFF}} \right) \left[ (V_{GS} - V_{TH})^2 \right] \left( 1 + (\lambda \cdot V_{DS}) \right)$$

Threshold Voltage:
$$V_{TH} = V_{TO} + \gamma \sqrt{2 \cdot \phi \cdot V_{BS} - \sqrt{2 \cdot \phi}}$$

Channel Length
$$L_{EFF} = L - 2LD$$
MOSFET Small Signal Model and Analysis

SPICE MOSFET Model – Additional Parameters

SPICE takes many of its parameters from the integrated circuit layout design:

\[
\begin{align*}
L &= \text{polysilicon gate length} \\
W &= \text{polysilicon gate width} \\
AD &= \text{drain area} \\
AS &= W \times L_{\text{diff}}(\text{source}) \\
PD &= 2 \times L_{\text{diff}}(\text{drain}) + W \\
PS &= 2 \times L_{\text{diff}}(\text{source}) + W \\
NRD &= \text{number of “squares” in drain diffusion} \\
NRS &= \text{number of “squares” in source diffusion}
\end{align*}
\]

Specified in terms of the minimum feature size.
## SPICE MOSFET Model – Additional Parameters

<table>
<thead>
<tr>
<th>Model Parameters (see .MODEL statement)</th>
<th>Default value</th>
<th>Units</th>
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<tbody>
<tr>
<td>LEVEL</td>
<td>model type (1, 2, or 3)</td>
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<tr>
<td>L</td>
<td>channel length</td>
<td>DEFL</td>
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<tr>
<td>W</td>
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- **Most Used Parameters**: Level 1, L, W, LD, WD, VTO, KP, GAMMA, PHI, LAMBDA, RD, RS, RG, RB, RDS, RSH, IS, PB, CBD, CBS, CJ, CJSW, MJ, MJSW, FC, CGSO, CGDO, CGBO, NSUB, NSS, NFS, TOX, TPG, X1, UO, UCRIT, UEXP, UTRA, VMAX, NEFF, XOC, DELTA, THETA, ETA, KAPPA, KF, AF.