Lecture 26

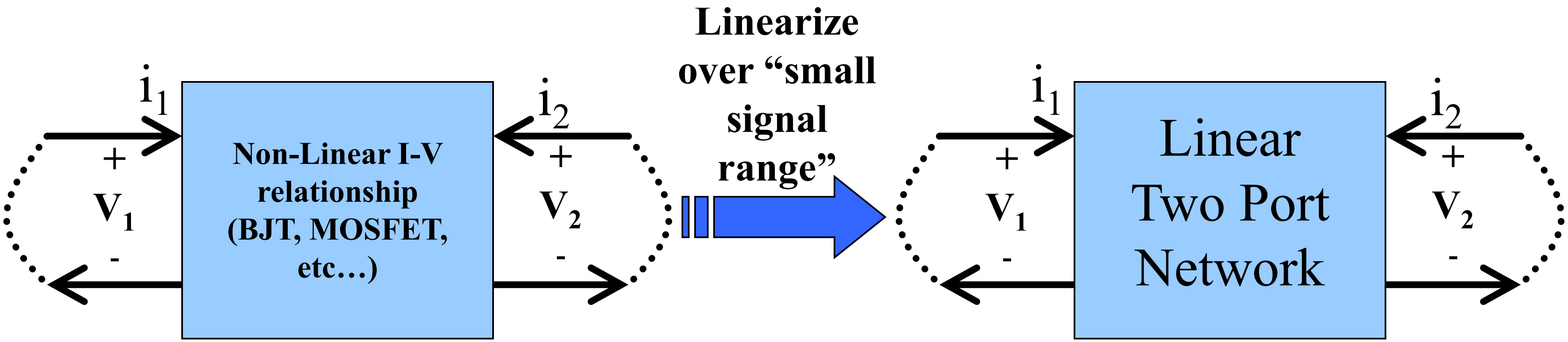
MOSFET Small Signal Model

Reading: Jaeger 13.7 and Notes

MOSFET Small Signal Model and Analysis

- Just as we did with the BJT, we can consider the MOSFET amplifier analysis in two parts:
- Find the DC operating point
- Then determine the amplifier output parameters for very small input signals.

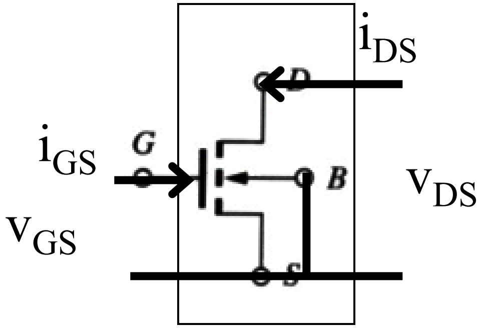
MOSFET Small Signal Model and Analysis



Non-Linear I-V relationship (BJT, MOSFET, etc…)

Linearize over "small signal range"

Linear Two Port Network



General "y-parameter" Network

$$I_1 = y_{11}V_1 + y_{12}V_2$$

$$I_2 = y_{21}V_1 + y_{22}V_2$$

MOSFET "y-parameter" Network

$$I_{GS} = y_{11}V_{GS} + y_{12}V_{DS}$$

$$I_{DS} = y_{21}V_{GS} + y_{22}V_{DS}$$

MOSFET Small Signal Model and Analysis

$$\begin{bmatrix} I_{GS} \\ I_{DS} \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_{GS} \\ V_{DS} \end{bmatrix}$$

$$I_{GS} = y_{11} V_{GS} + y_{12} V_{DS}$$

$$I_{DS} = y_{21} V_{GS} + y_{22} V_{DS}$$

$$y_{ij} = \left. \frac{\partial I_j}{\partial V_i} \right|_{V_{GS},Q\,,\,V_{DS},Q}$$

Derivative of current-voltage equation evaluated at the Quiescent Point

MOSFET Amplifiers are biased into Saturation (or Active Mode)

$$I_{DS} = \frac{K_n}{2}\left[(V_{GS} - V_{TN})^2\right](1 + \lambda \; V_{DS}) \quad for \quad V_{DS} \geq V_{GS} - V_{TN}$$

1.) Input Conductance

$$I_{GS} = 0 \quad \Rightarrow \quad \boxed{\frac{\partial I_{GS}}{\partial V_{GS}} = 0 \quad and \quad \frac{\partial I_{GS}}{\partial V_{DS}} = 0} \quad \Rightarrow \quad \boxed{y_{11} = 0 \; and \; y_{12} = 0}$$

2.) Output Conductance

$$\frac{\partial I_{DS}}{\partial V_{DS}} = \boxed{y_{22} = \frac{\lambda \, K_n}{2}(V_{GS} - V_T)^2}$$

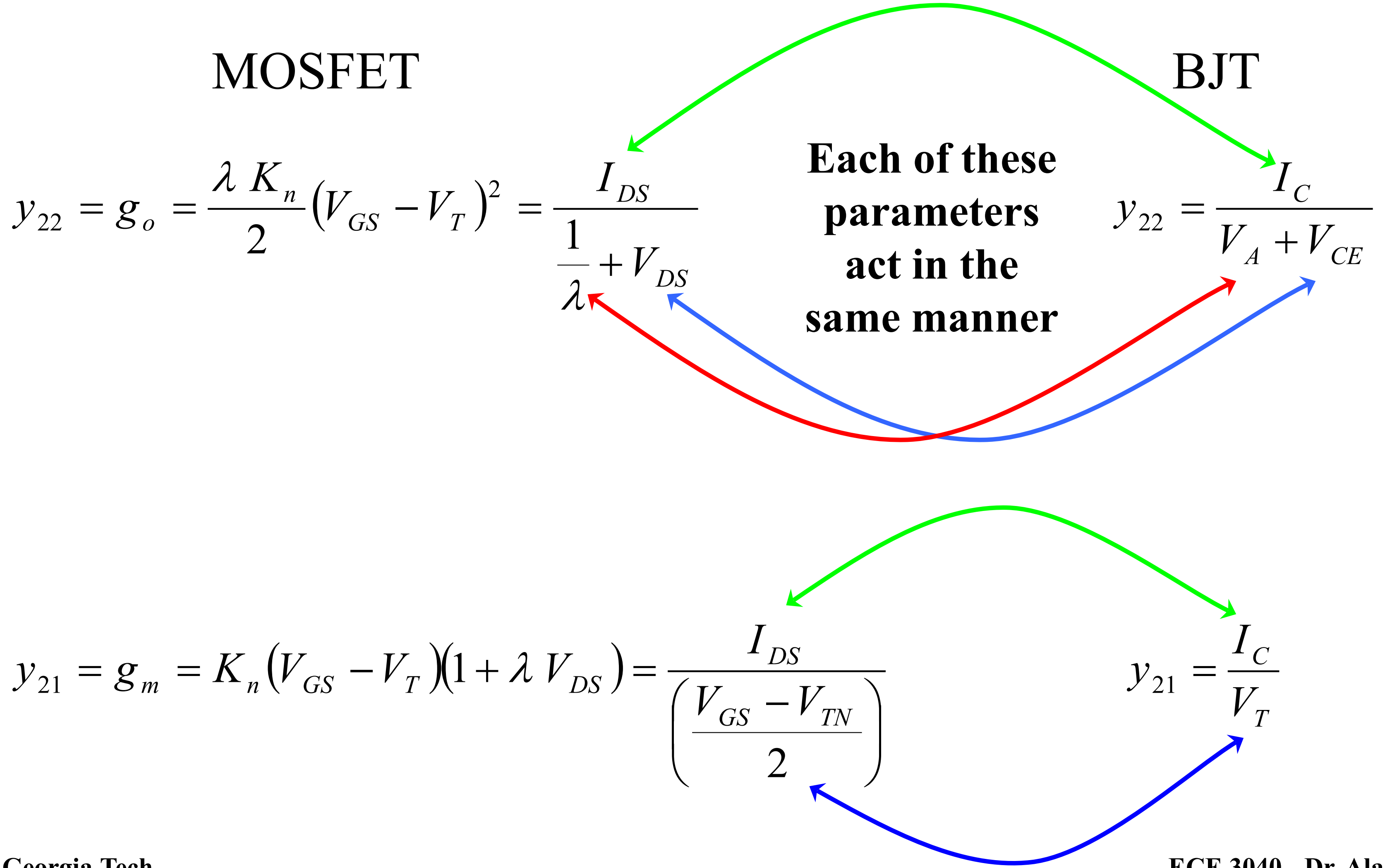
3.) Transconductance

$$\frac{\partial I_{DS}}{\partial V_{GS}} = \boxed{y_{21} = K_n (V_{GS} - V_T)(1 + \lambda \, V_{DS})}$$

MOSFET Small Signal Model and Analysis

Compare with BJT Results

There is a large amount of symmetry between the MOSFET and the BJT



MOSFET

BJT

$$y_{22} = g_o = \frac{\lambda K_n}{2}\left(V_{GS}-V_T\right)^2 = \frac{I_{DS}}{\frac{1}{\lambda}+V_{DS}}$$

Each of these parameters act in the same manner

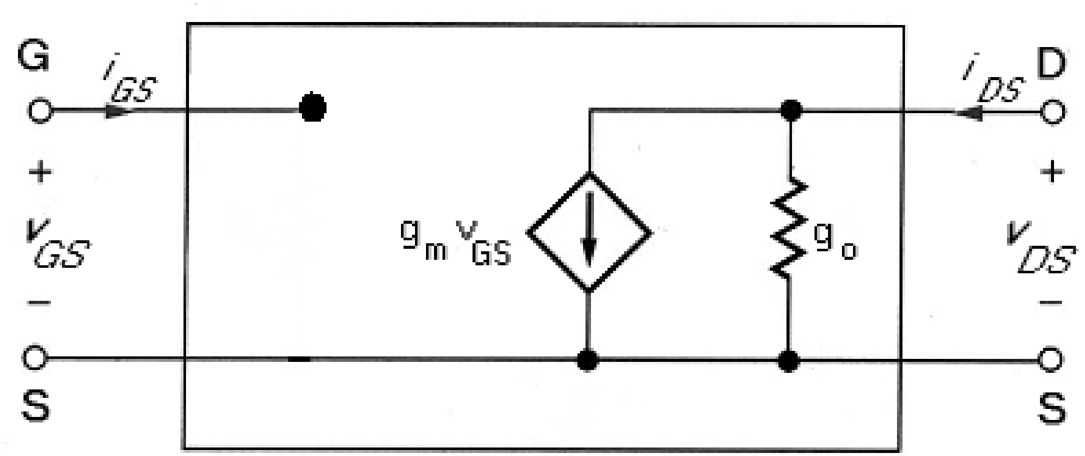
$$y_{22} = \frac{I_C}{V_A+V_{CE}}$$

$$y_{21} = g_m = K_n\left(V_{GS}-V_T\right)\left(1+\lambda V_{DS}\right) = \frac{I_{DS}}{\left(\frac{V_{GS}-V_{TN}}{2}\right)}$$

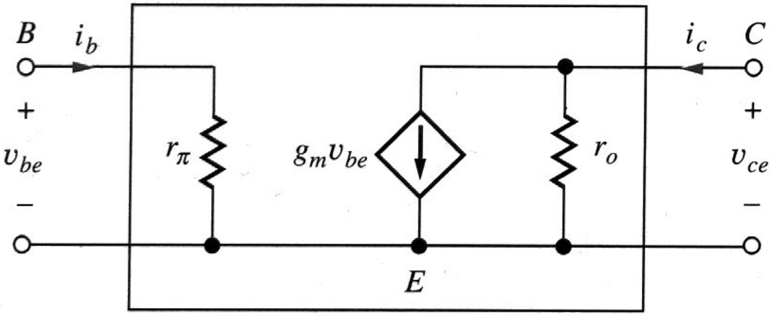
$$y_{21} = \frac{I_C}{V_T}$$

MOSFET Small Signal Model and Analysis

Putting the mathematical model into a small signal equivalent circuit

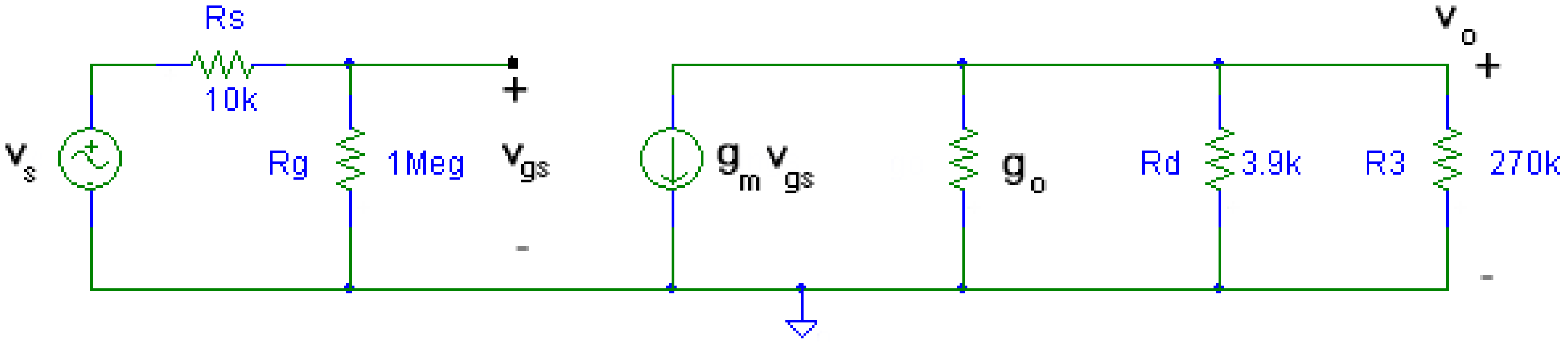


Compare this to the BJT small signal equivalent circuit



MOSFET Small Signal Model and Analysis

Example: Jaeger 13.94



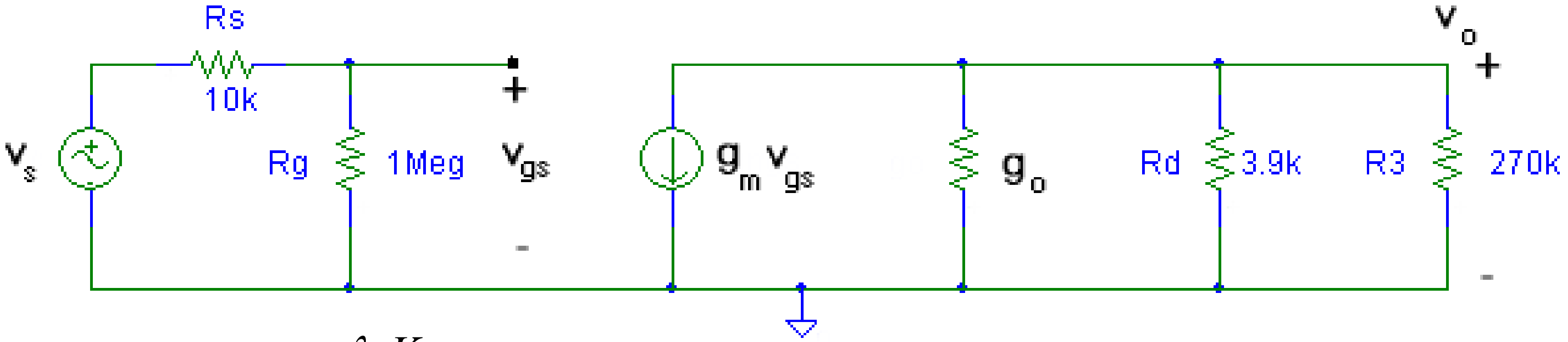
Calculate the voltage gain, $A_v = v_o / v_s$

Given: $K_n = 1\ mA/V^2$, $\lambda = 0.015\ V^{-1}$

Bias Point of: $I_{DS} = 2\ mA$, $V_{DS} = 7.5V$

MOSFET Small Signal Model and Analysis

Example: Jaeger 13.94



$$g_o = \frac{\lambda K_n}{2}(V_{GS} - V_T)^2 \qquad g_m = K_n(V_{GS} - V_T)(1 + \lambda V_{DS})$$

Need to find V_{GS}-V_T

$$I_{DS} = \frac{K_n}{2}\left[(V_{GS} - V_{TN})^2\right](1 + \lambda \ V_{DS})$$

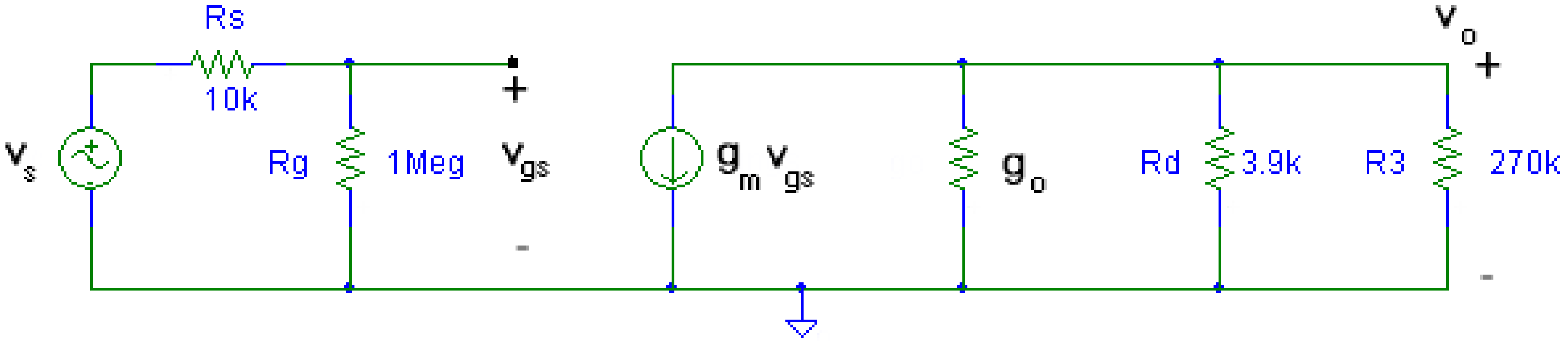
$$2\ mA = \frac{1\ mA/V^2}{2}\left[(V_{GS} - V_{TN})^2\right](1 + 0.015\ (7.5))$$

$$V_{GS} - V_{TN} = \sqrt{\frac{4}{1.11}} = 1.9V$$

$$\therefore g_m = 2.11\ mS \quad g_o = 27.1\ \mu\ S \Rightarrow r_o = 36.9k\Omega$$

MOSFET Small Signal Model and Analysis

Example: Jaeger 13.94



$$A_v = \frac{v_o}{v_s} = \frac{v_{GS}}{v_s}\frac{v_o}{v_{GS}}$$

$$\frac{v_{GS}}{v_s} = \frac{1Meg}{10k + 1Meg} = 0.99 \quad and \quad \frac{v_o}{v_{GS}} = -g_m\left(r_o \,||\, Rd \,||\, R3\right) = -2.1mS\left(3.48k\right) = -7.35$$

$$\therefore \quad A_v = \frac{v_o}{v_s} = \frac{v_{GS}}{v_s}\frac{v_o}{v_{GS}} = -7.27 \ \left[V/V\right]$$

MOSFET Amplifiers

Note source is the terminal tied to the body connection

$V_{SD}>V_{SG}-V_{TP}$

What is the Maximum Gain Possible?

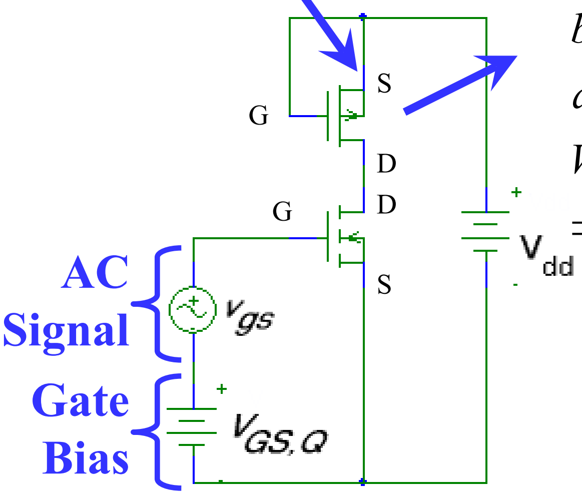
Is it saturated (Constant current), $|V_{DS}| \geq |V_{GS}-V_{TP}| \geq 0$

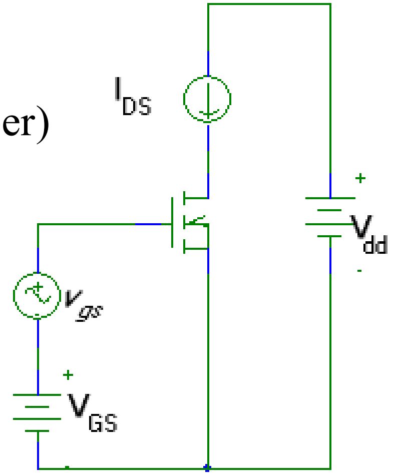
but $V_{GS}=0$

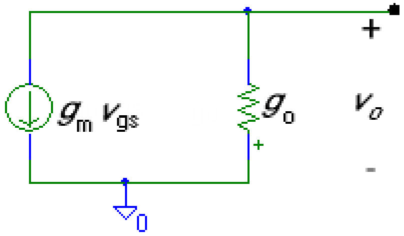
and $V_{TP} \geq 0$ for a depletion mode MOSFET *so*,

V_{DS} (a positve number) $>$ ($-V_{TP}$ a negative number)

$\Rightarrow$ Is Saturated!







g$_o$ is internal to the transistor and can not be avoided. Any additional resistor due to external circuitry will lower the gain. For this reason current sources are often used as the “load” instead of bias resistors in amplifier circuits.

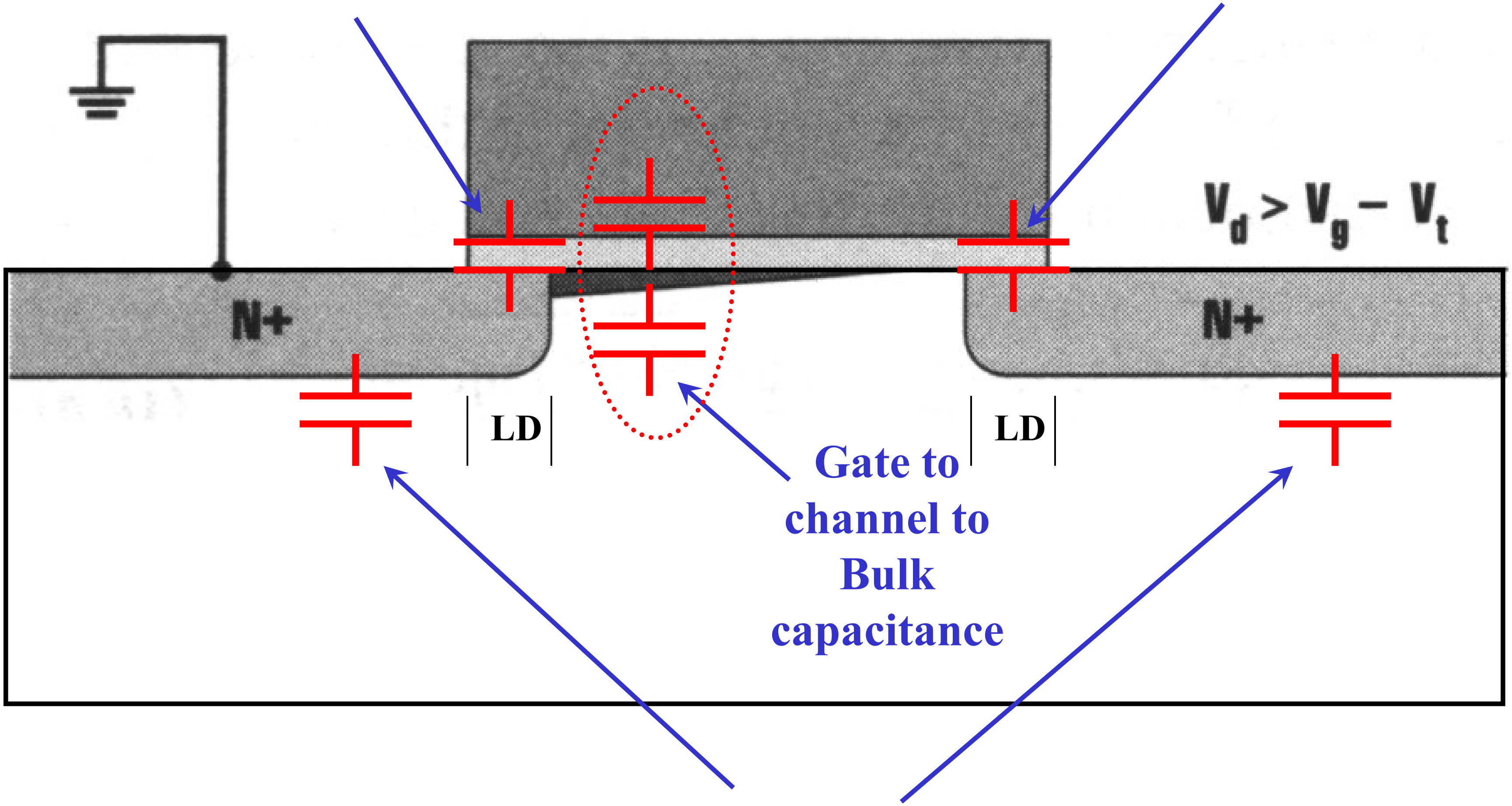
$$A_{v,Max}=-g_m r_o$$

$$A_{v,Max}=-\frac{K_n\left(V_{GS}-V_T\right)\left(1+\lambda\, V_{DS}\right)}{\lambda\, K_n\left(V_{GS}-V_T\right)^2}$$

$$A_{v,Max}=-\frac{\left(1+\lambda\, V_{DS}\right)}{\lambda\left(V_{GS}-V_T\right)}$$

MOSFET Small Signal Model and Analysis

Add in capacitances



Overlap of Gate Oxide

Overlap of Gate Oxide

Gate to channel to Bulk capacitance

Reverse Bias Junction capacitances

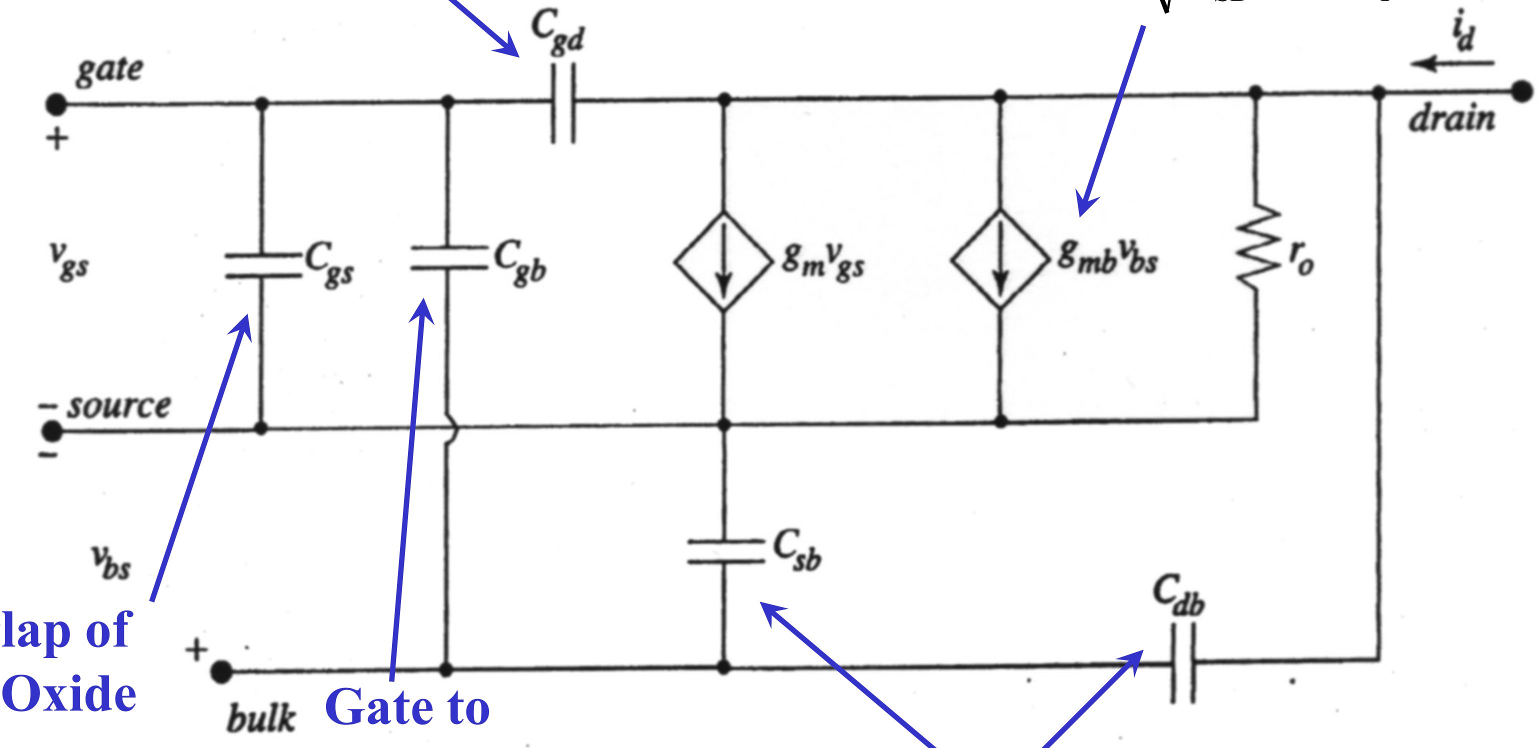
MOSFET Small Signal Model and Analysis

Complete Model of a MOSFET

Overlap of Gate Oxide

$$g_{mb} = g_m \frac{\gamma}{2\sqrt{V_{SB}+2\phi_F}}$$

Due to effective modulation of the threshold voltage.



Overlap of Gate Oxide and source

Gate to channel to Bulk capacitance

Reverse Bias Junction capacitances

MOSFET Small Signal Model and Analysis

SPICE MOSFET Model

SPICE models the drain current (I_{DS}) of an n-channel MOSFET using the following parameters/equations (SPICE variables are shown in ALL CAPPITAL LETTERS)

Cutoff: $I_{DS} = 0$

Linear:

$$I_{DS} = \frac{KP}{2}\left(\frac{W}{L_{EFF}}\right)V_{DS}\left[2\left(V_{GS} - VTH\right) - V_{DS}\right]\left(1 + \left(LAMBDA\right)V_{DS}\right)$$

Saturation:

$$I_{DS} = \frac{KP}{2}\left(\frac{W}{L_{EFF}}\right)\left[\left(V_{GS} - VTH\right)^2\right]\left(1 + \left(LAMBDA\right)V_{DS}\right)$$

Threshold Voltage:

$$V_{TH} = VTO + GAMMA\left(\sqrt{2PHI - V_{BS}} - \sqrt{2PHI}\right)$$

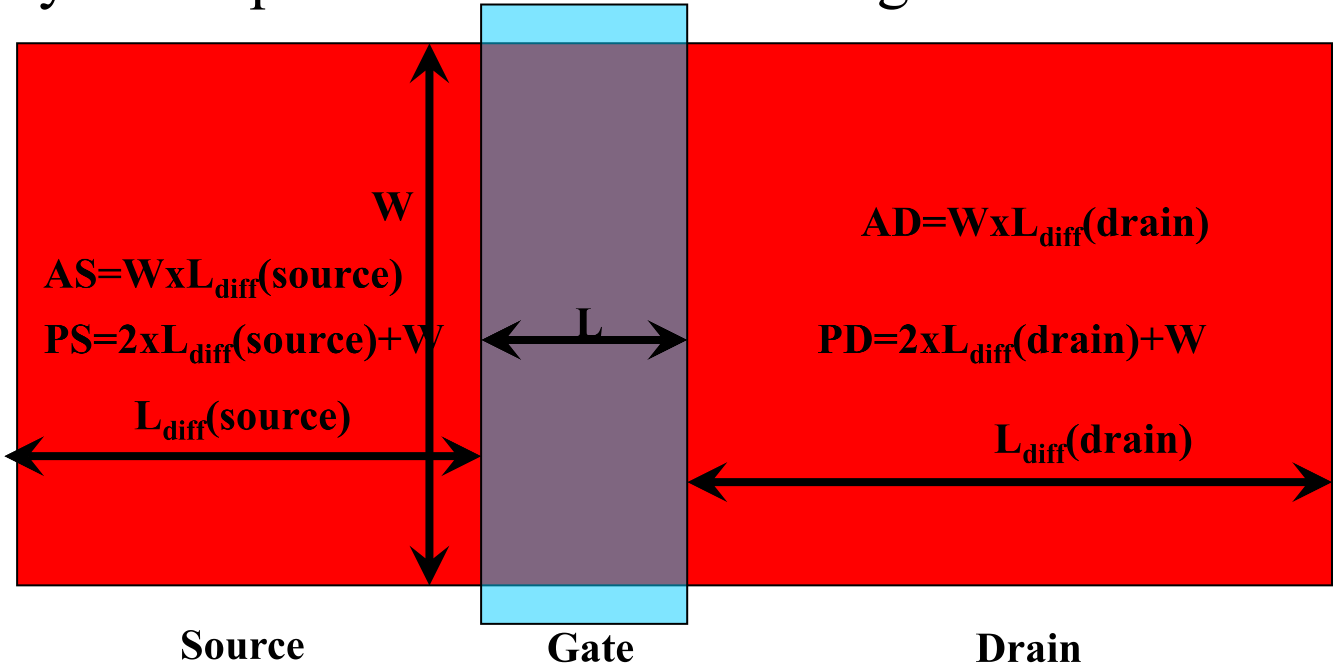
Channel Length

$$L_{EFF} = L - 2LD$$

MOSFET Small Signal Model and Analysis

SPICE MOSFET Model – Additional Parameters

SPICE takes many of it’s parameters from the integrated circuit layout design:



L = polysilicon gate length

W = polysilicon gate width

AD = drain area

AS = source area

PD = perimeter of drain diffusion (not including edge under gate)

PS = perimeter of source diffusion (not including edge under gate)

NRD = number of “squares” in drain diffusion

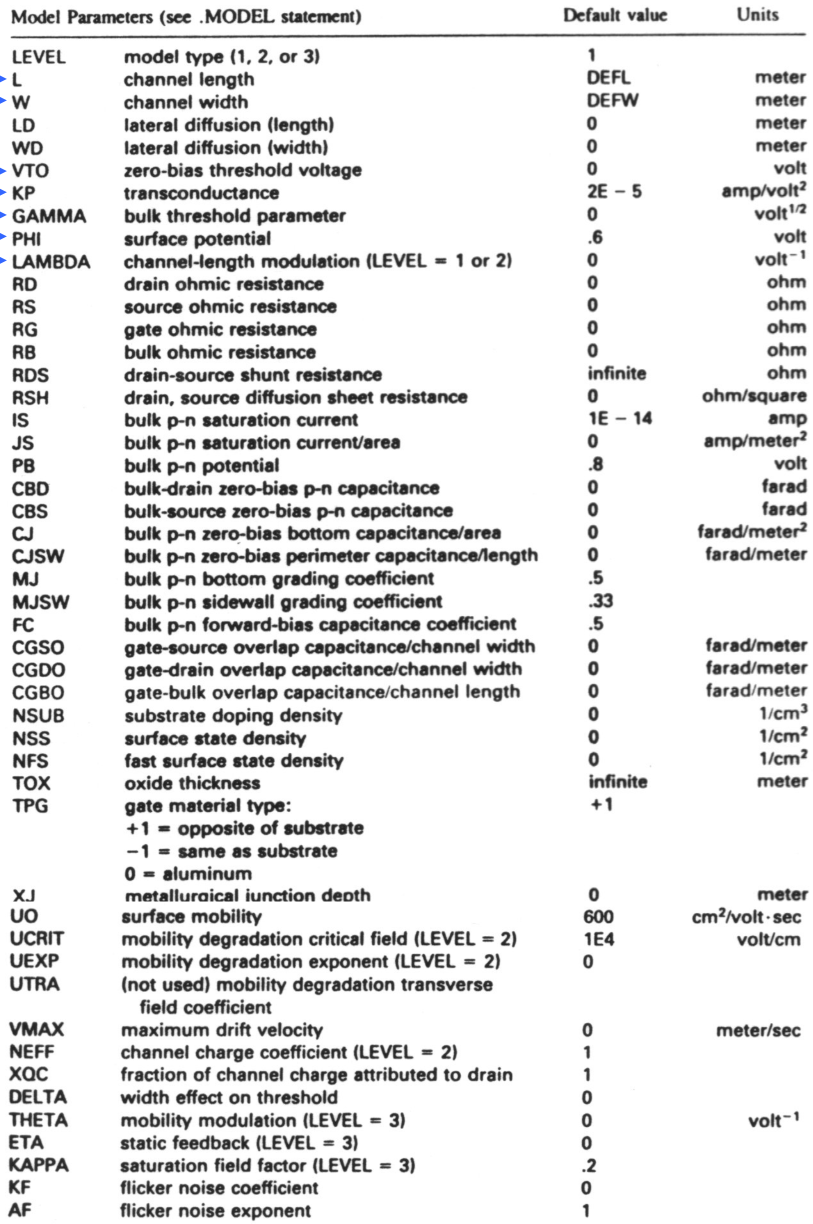
NRS = number of “squares” in source diffusion

} Specified in terms of the minimum feature size

MOSFET Small Signal Model and Analysis

SPICE MOSFET Model – Additional Parameters

Most Used



Model Parameters (see .MODEL statement)		Default value	Units
LEVEL	model type (1, 2, or 3)	1	
L	channel length	DEFL	meter
W	channel width	DEFW	meter
LD	lateral diffusion (length)	0	meter
WD	lateral diffusion (width)	0	meter
VTO	zero-bias threshold voltage	0	volt
KP	transconductance	2E − 5	amp/volt^2
GAMMA	bulk threshold parameter	0	volt$^{1/2}$
PHI	surface potential	.6	volt
LAMBDA	channel-length modulation (LEVEL = 1 or 2)	0	volt^{-1}
RD	drain ohmic resistance	0	ohm
RS	source ohmic resistance	0	ohm
RG	gate ohmic resistance	0	ohm
RB	bulk ohmic resistance	0	ohm
RDS	drain-source shunt resistance	infinite	ohm
RSH	drain, source diffusion sheet resistance	0	ohm/square
IS	bulk p-n saturation current	1E − 14	amp
JS	bulk p-n saturation current/area	0	amp/meter^2
PB	bulk p-n potential	.8	volt
CBD	bulk-drain zero-bias p-n capacitance	0	farad
CBS	bulk-source zero-bias p-n capacitance	0	farad
CJ	bulk p-n zero-bias bottom capacitance/area	0	farad/meter^2
CJSW	bulk p-n zero-bias perimeter capacitance/length	0	farad/meter
MJ	bulk p-n bottom grading coefficient	.5	
MJSW	bulk p-n sidewall grading coefficient	.33	
FC	bulk p-n forward-bias capacitance coefficient	.5	
CGSO	gate-source overlap capacitance/channel width	0	farad/meter
CGDO	gate-drain overlap capacitance/channel width	0	farad/meter
CGBO	gate-bulk overlap capacitance/channel length	0	farad/meter
NSUB	substrate doping density	0	1/cm^3
NSS	surface state density	0	1/cm^2
NFS	fast surface state density	0	1/cm^2
TOX	oxide thickness	infinite	meter
TPG	gate material type: +1 = opposite of substrate −1 = same as substrate 0 = aluminum	+1	
XJ	metallurgical junction depth	0	meter
UO	surface mobility	600	cm^2/volt·sec
UCRIT	mobility degradation critical field (LEVEL = 2)	1E4	volt/cm
UEXP	mobility degradation exponent (LEVEL = 2)	0	
UTRA	(not used) mobility degradation transverse field coefficient		
VMAX	maximum drift velocity	0	meter/sec
NEFF	channel charge coefficient (LEVEL = 2)	1	
XQC	fraction of channel charge attributed to drain	1	
DELTA	width effect on threshold	0	
THETA	mobility modulation (LEVEL = 3)	0	volt^{-1}
ETA	static feedback (LEVEL = 3)	0	
KAPPA	saturation field factor (LEVEL = 3)	.2	
KF	flicker noise coefficient	0	
AF	flicker noise exponent	1	