**3.1. Calculate the speed of an electron in Si with kinetic energy 0.013 eV. Draw an equilibrium energy band diagram for silicon and indicate where this electron will be. Compare your calculated thermal speed to the typical drift velocities cited in the text of $10^4$ cm/s. How does it compare to typical saturation velocities?**

The electron speed is

$$v = \sqrt{\frac{2E_K}{m_{ce}^*}} = \sqrt{\frac{2\left(0.013eV \cdot 1.6 \times 10^{-19} J/eV\right)}{0.26\left(9.1 \times 10^{-31} kg\right)}} = 1.3 \times 10^5 \, m/s$$

$$= 1.3 \times 10^5 \, m/s \times \frac{100cm}{m} = 1.3 \times 10^7 \, cm/s$$

Even with this very small kinetic energy, the electron's instantaneous speed is still about 1000 times faster than the drift velocity. It is comparable to the drift saturation velocity for silicon of $1 \times 10^7$ cm/s.
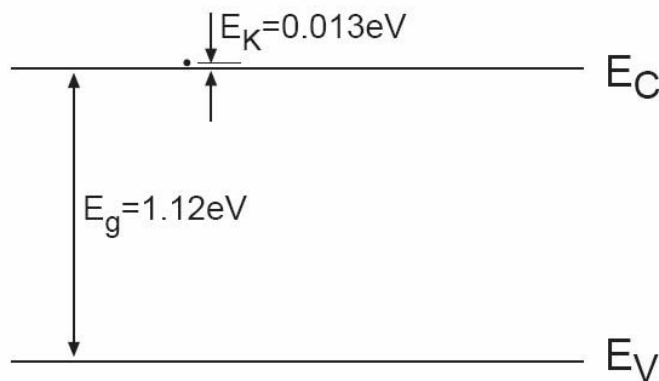
**3.6. A voltage of 2.5V is applied to a sample of silicon whose cross-sectional area is 0.1 μm ×1 μm. The length of the path is 0.1 μm. If the material is doped n-type with $N_D=10^{18}$ cm$^{-3}$, what is the current in the sample? What is the current density? These dimensions could represent the channel of a field effect transistor.**

The cross sectional area is $A = 0.1\mu m \times 1.0 \mu m = 10^{-9} cm^2$, and the length is $L=0.1\mu m=10^{-5}$ cm..

We know the current is given by $I=JA$, and $J=\sigma \mathcal{E}$. To find $\sigma$, we need to find $n_0$ and $p_0$. We can write that $n_0=N_D=10^{18}$, and $p_0$ will be negligible so it won't contribute to conductivity. From Figure 3.4 for silicon doped to $10^{18}$cm$^{-3}$, we find that the majority carrier mobility $\mu_n \approx 230$ cm$^2$/V·s, and

$$\sigma = q\mu_n n = \left(1.6 \times 10^{-19} C\right)(230)\left(10^{18}\right)= 37(\Omega - cm)^{-1}$$

Thus

$$J = \sigma \mathrm{E} = \sigma \frac{V}{L} = 37\left(\Omega - cm\right)^{-1} \cdot \frac{2.5V}{0.1 \times 10^{-4} cm} = 9 \times 10^6 A/cm^2$$

and.

$$I = JA = 9 \times 10^6 \, Amps/cm^2 \cdot 10^{-9} cm^2 = 9mA$$

**3.11. Compare the mean free time between collisions for electrons and for holes in intrinsic GaAs. How do these values compare to those for silicon?**

Following the method of Example 3.2, we have

$$\bar{t}_n = \frac{m_{ce}^*\mu_n}{q} = \frac{(0.067m_0)\left(8\times10^3 cm^2 / \ V\cdot s\times\dfrac{1m^2}{10^4 cm^2}\right)}{1.6\times10^{-19}C} = \frac{0.067\left(9.1\times10^{-31}kg\right)\left(0.8m^2 / \ V\cdot s\right)}{1.6\times10^{-19}C}$$

$$= 0.3ps$$

where we obtained the value of $\mu_n$ from Figure 3.7 and $m_{ce}^*$ from the table inside the book's cover. This is comparable to the case for silicon (0.2ps).

For holes,

$$\bar{t}_p = \frac{m_{ch}^*\mu_p}{q} = \frac{(0.34m_0)\left(4\times10^2 cm^2 /V\cdot s\times\dfrac{1m^2}{10^4 cm^2}\right)}{1.6\times10^{-19}C} = 7.7\times10^{-14}s = 0.077\,ps$$

compared to 0.1 ps in silicon.

**3.12. For the case of Problem 3.11, find an average gain in kinetic energy between collisions for electrons and holes for an applied field of 100 V/cm.**

The electrons attain an average velocity of (Equation (3.27))

$$v_{dn} = -\frac{qE}{m_{ce}^*}\bar{t}_n = -\frac{\left(1.6\times10^{-19}C\right)\left(100V/cm\right)\left(\dfrac{100cm}{1m}\right)}{\left(0.067\right)\left(9.1\times10^{-31}kg\right)}\left(0.3\times10^{-12}s\right) = 7.9\times10^3\,m/s$$

The corresponding kinetic energy is

$$E_K = \frac{1}{2}m_{ce}^*v_{dn}^2 = \frac{(0.067)(9.1\times10^{-31}kg)}{2}\left(7.9\times10^3\,m/s\right)^2$$

$$= 1.9\times10^{-24}\,J\left(\frac{1J}{1.6\times10^{-19}eV}\right)$$

$$= 1.2\,\mu eV$$

For holes the result is

$$v_{dp} = \frac{qE}{m_{ch}^*}\bar{t}_p = \frac{\left(1.6\times10^{-19}C\right)\left(100\dfrac{V}{cm}\right)\left(\dfrac{100cm}{m}\right)}{\left(0.34\right)\left(9.1\times10^{-31}kg\right)}7.70\times10^{-14}s$$

$$= 4.0\times10^2\,m/s$$

and the kinetic energy is

$$E_K = \frac{1}{2}m_{ch}^*v_{dp}^2 = \frac{(0.34)(9.1\times10^{-31}kg)}{2}\left(4.0\times10^6\,m/s\right)^2$$

$$= 2.5\times10^{-26}\,J\left(\frac{1J}{1.6\times10^{-19}eV}\right)$$

$$= 0.15\,\mu eV$$

**3.13. Estimate the saturation velocity of electrons in intrinsic GaAs. How does your estimate compare with the experimental data?**

Following Example 3.3, we make the assumptions again that an electron loses all of its kinetic energy after each collision, that an electron gains an energy equal to $E_{pho}$ between collisions, and the electrons remain in a region of constant and valid effective mass.

From the table inside the book cover, $E_{pho}$ for GaAs is 0.034 eV, and we have from Equation (3.35),

$$v_{sat} = \frac{v_{max}}{2} = \sqrt{\frac{E_{pho}}{2m_{ce}^*}} = \sqrt{\frac{0.034ev(1.6\times10^{-19}\,J/eV)}{2(0.067)(9.1\times10^{-31}\,kg)}} \cdot \left(\frac{100cm}{m}\right) = 2.1\times10^7\,cm/s$$

From Figure 3.9, we see that the saturation velocity for GaAs peaks at about this value, $2\times10^7$ (a very good match), but then decreases and levels off at about $6\times10^6$. The reduction is due to scattering into the other conduction band minima, invalidating our assumption that the electron remains in a region in which the concept of effective mass is valid- to go from one such region to another involves crossing through a region in which this assumption is not true.

$$J_{n(diff)} = qD_n\frac{dn(x)}{dx} = \left(1.6\times10^{-19}\,C\right)\left(9\frac{cm^2}{s}\right)\left(-10^7\,cm^{-4}\right) = -0.014nA/cm^2$$

The minus sign indicates that the current is flowing from a region of higher concentration to a region of lower concentration.

**3.15. Consider a p-type Si sample of $N_A = 10^{18}$cm$^{-3}$ and $N_D = 0$. Over a length of 1 μm the electron concentration drops linearly from $10^{16}$cm$^{-3}$ to $10^{13}$cm$^{-3}$. Calculate the electron diffusion current density.**

From Equation (3.40), we have

$J_{n(diff)} = qD_n \dfrac{dn(x)}{dx}$. Since the electron concentration is dropping linearly, the expression for n(x) is

$n(x) = -\dfrac{10^3 \, cm^{-3}}{10^{-4} \, cm} x$, and the derivative is

$\dfrac{dn}{dx} = -10^{-7} cm^{-4}$.

At $N_A = 10^{18}$cm$^{-3}$, the diffusion constant is, from Figure 3.11 (electrons are minority carriers since the material is p-type), $D_n = 9$cm$^2$/s. Thus

3.19. Light of $h\nu$=1.5eV (this in the near infrared, at $\lambda$=826 nm) at a power level of 10 mW shines on an intrinsic sample of GaAs of area 1 cm². Let the electron lifetime be 10 ps.

a) What is the number of photons arriving at the semiconductor surface per sec? (recall energy=power×time).

One photon has energy of 1.5 eV, so to produce a power of 10 mW, we require

$$10mW = \frac{N(photons) \times 1.5eV / \ photon \times \left(1.6 \times 10^{-19} J / \ eV\right)}{1s}$$

$$N = 4.2 \times 10^{16} photons / \ s$$

b) Verify that photons at this energy can be absorbed.

For the photons to be absorbed, the energy of a photon must be greater than the bandgap. The energy gap of GaAs is 1.43 eV, so these photons can be absorbed.

**c) Assuming every photon is absorbed and creates an electron-hole pair, and assuming the GaAs sample is 1 mm thick, what is the optical generation rate?**

There are $N=4.2\times10^{16}$ photons/sec arriving on an area 1 cm$^2$, and absorbed in a volume 1 cm$^2$x0.1cm=0.1 cm$^3$. Since each photon produces and electron and a hole, the average optical generation rate is

$$\langle G_{op} \rangle = \frac{N}{thickness} = \frac{4.2\times10^{16}\,photons\,/\,s\,/\,cm^2}{0.1cm} = 4.2\times10^{17}\,\text{e-h pairs/sec/cm}^3$$

**d) What are the equilibrium electron and holes densities (in the dark)?**

Since the GaAs is intrinsic, $n_0 = p_0 = n_i = 2.2\times10^6$ cm$^{-3}$.

**e) What are the excess carrier concentrations when the light is on?**

Since this is steady state, $\dfrac{dn}{dt}$ =0. From Equation (3.63) we have

$$\frac{\partial n}{\partial t} = \frac{1}{q}\left(\frac{\partial J_n}{\partial x}\right) + \left(G_{op} - \frac{\Delta n}{\tau_n}\right) = 0$$

If the photons are uniformly absorbed, there is no diffusion current, and

$$\Delta n = G_{op}\tau_n = 4.2\times10^{17}\,cm^{-3}s^{-1}\left(10\times10^{-12}\,s\right) = 4.2\times10^6\,cm^{-3}$$

**f) What are the recombination rates for electrons and holes when the light is off? When the light is on?**

From Equation (3.60),

$$R = \frac{n_0}{\tau_n} + \frac{\Delta n}{\tau_n}$$

When the light is off, $n_0 = n_i$ (since this is intrinsic GaAs), or $n_0 = 2.2\times10^6$ cm$^{-3}$ and $\Delta n = 0$.

$$R_{off} = \frac{n_0}{\tau_n} = \frac{2.2\times10^6\,cm^{-3}}{10\times10^{-12}\,s} = 2.2\times10^{17}\,cm^{-3}s^{-1}$$

When the light is on,

$$R_{on} = \frac{n_0}{\tau_n} + \frac{\Delta n}{\tau_n} = 2.2 \times 10^{17} + \frac{4.2 \times 10^6\, cm^{-3}}{10 \times 10^{-12}} = 2.2 \times 10^{17} + 4.2 \times 10^{17} = 6.4 \times 10^{17}\, cm^{-3} s^{-1}$$

**g. What are the steady-state carrier densities $n$ and $p$?**

For electrons we have
$n=n_0+\Delta n=2.2\times10^6$ cm$^{-3}$ + 4.2$\times10^6$ cm$^{-3}$=6.4$\times10^6$ cm$^{-3}$

Since the material is intrinsic, $n_0=p_0=n_i$, and since the excess electrons and holes are created in pairs, $\Delta n=\Delta p$, so
$p=p_0+\Delta p=n_0+\Delta n=n=6.4\times10^6$ cm$^{-3}$ .

**h. How much does the conductivity of this sample change compared with its**

**dark value?**

We know that

$$\sigma = q\left(\mu_n n + \mu_p p\right) = q\left(\mu_n n_0 + \mu_p p_0\right) + q\left(\mu_n \Delta n + \mu_p \Delta p\right)$$

The mobilities for intrinsic GaAs are, from Figure 3.7, $\mu_n$=8000 cm$^2$/V-s and $\mu_p$=400 cm$^2$/V-s. In the dark, the conductivity is

$$\sigma = q\left(\mu_n n_0 + \mu_p p_0\right) = 1.6 \times 10^{-19}\, C\left[\left(8000 cm^2 / V \cdot s\right)\left(2.2 \times 10^6\, cm^{-3}\right) + \left(400 cm^2 / V \cdot s\right)\left(2.2\right.\right.$$

$$= 3.0 \times 10^{-9} \left(\Omega - cm\right)^{-1}$$
When the light is on, the result is

$$\sigma = q\left(\mu_n n + \mu_p p\right)$$

$$= 1.6 \times 10^{-19}\, C\left[\left(8000 cm^2 / V \cdot s\right)\left(6.4 \times 10^6\, cm^{-3}\right) + \left(400 cm^2 / V \cdot s\right)\left(6.4 \times 10^6\, cm^{-3}\right)\right]$$

$$= 8.6 \times 10^{-9} \left(\Omega - cm\right)^{-1}$$
showing that the conductivity nearly triples when the light is on in this case.

**i) Suppose the power level is kept the same, but the wavelength of the light is shifted further into the infrared, at $E=h\nu=1$ eV ($\lambda=1240$ nm). What is the generation rate now?**

It is zero. These photons have energy smaller than the band gap and cannot be absorbed.

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**3.26. A sample of InP is doped such that $E_f - E_V = 0.2$ eV. It is also illuminated such that $\Delta n = \Delta p = 10^3$cm$^{-3}$. Find the quasi-Fermi levels and sketch the energy band diagram. Repeat for $\Delta n = \Delta p = 10^{10}$.**

We find the equilibrium carrier concentrations:

$$p_0 = N_V e^{-\left(\frac{E_f - E_V}{kt}\right)} = 6.8 \times 10^{18}\, cm^{-3} e^{\frac{0.2}{0.026}} = 3.1 \times 10^{15}\, cm^{-3}$$

$$n_0 = \frac{n_i^2}{p_0} = \frac{\left(1.97 \times 10^7\right)^2}{3.1 \times 10^{15}} = 0.12\, cm^{-3}$$

The total carrier concentrations are:

$$n = n_0 + \Delta n = 0.12 + 10^3 \approx 10^3\, cm^{-3}; \quad p = p_0 + \Delta p = 3.1 \times 10^{15} + 10^3 \approx 3.1 \times 10^{15}\, cm^{-3}$$

From which we can find the quasi -Fermi levels:

$$E_C - E_{fn} = -kT \ln\left(\frac{n}{N_C}\right) = -0.026eV \cdot \ln\left(\frac{10^3}{2.07 \times 10^{18}}\right) = 0.91eV, \text{ and } E_{fp} \text{ should}$$

be the same as $E_f$ since the majority carrier concentration is almost unaffected.

When the excess carrier concentrations are $10^{10}$, $n=n_0+\Delta n\approx 10^{10}$cm$^{-3}$, and

$$E_C - E_{fn} = -kT \ln\left(\frac{n}{N_C}\right) = -0.026eV \cdot \ln\left(\frac{10^{10}}{2.07 \times 10^{18}}\right) = 0.50eV.$$ The excess

hole concentration is still negligible compared to the equilibrium concentration of holes and so $E_{fp}$ is still equal to $E_f$.

The energy band diagram for both cases is shown below:

0.91 eV    0.5 eV

$E_C$

1.43 eV

$E_{fn}$ ($\delta n = 10^{10}$)

$E_i$   $E_{fn}$ ($\delta n = 10^3$)

$E_f = E_{fp}$

$E_V$

0.2 eV