Lecture 5

Carrier Concentrations in Equilibrium

Reading:

(Cont'd) Notes and Anderson[2] sections from lecture 4

Developing the mathematical model for electrons and holes

Motivation:

Since current (electron and hole flow) is dependent on the concentration of electrons and holes in the material, we need to develop equations that describe these concentrations.

Furthermore, we will find it useful to relate the these concentrations to the average energy (fermi energy) in the material.

Developing the mathematical model for electrons and holes

The density of electrons is:

Probability the state is filled

$$n = \int_{E_{Bottom\ of\ conduction\ band}}^{E_{Top\ of\ conduction\ band}} g_c(E) f(E) dE$$

Number of states per cm^{-3} in energy range dE

The density of holes is:

Probability the state is empty

$$p = \int_{E_{Bottom\ of\ valence\ band}}^{E_{Top\ of\ valence\ band}} g_v(E)[1 - f(E)] dE$$

Number of states per cm^{-3} in energy range dE

Note: units of n and p are #/cm^3

Developing the mathematical model for electrons and holes

$$n = \frac{m_n^* \sqrt{2m_n^*}}{\pi^2 \hbar^3} \int_{E_c}^{E_{Top\ of\ conduction\ band}} \frac{\sqrt{E - E_c}}{1 + e^{(E - E_f)/kT}} dE$$

$$Letting \quad \eta = \frac{E - E_c}{kT} \quad and \quad \eta_c = \frac{E_f - E_c}{kT}$$

$$when\ E = E_c, \eta = 0$$

$$Let \quad E_{Top\ of\ conduction\ band} \rightarrow \infty$$

$$n = \frac{m_n^* \sqrt{2m_n^*} (kT)^{3/2}}{\pi^2 \hbar^3} \underbrace{\int_0^\infty \frac{\eta^{1/2}}{1 + e^{(\eta - \eta_c)}} d\eta}$$

This is known as the Fermi-dirac integral of order 1/2 or, $F_{1/2}(\eta_c)$

Developing the mathematical model for electrons and holes

We can further define:

$$N_c = 2\left[\frac{m_n^*(kT)}{2\pi\hbar^2}\right]^{3/2} \quad \textit{the effective density of states in the conduction band}$$

and

$$N_v = 2\left[\frac{m_p^*(kT)}{2\pi\hbar^2}\right]^{3/2} \quad \textit{the effective density of states in the valence band}$$

This is a general relationship holding for all materials and results in:

$$N_c = 2.51x10^{19}\left(\frac{m_n^*}{m_o}\right)^{3/2} cm^{-3} \quad at\ 300K$$

$$N_v = 2.51x10^{19}\left(\frac{m_v^*}{m_o}\right)^{3/2} cm^{-3} \quad at\ 300K$$

Developing the mathematical model for electrons and holes

$$n = N_c \frac{2}{\sqrt{\pi}} \underbrace{F_{1/2}(\eta_c)}$$

and

$$p = N_v \frac{2}{\sqrt{\pi}} \underbrace{F_{1/2}(\eta_v)} \quad where\ \eta_v = (E_v - E_f)/kT$$

Fermi-dirac integrals can be numerically determined or read from tables or...

Developing the mathematical model for electrons and holes

Useful approximations to the Fermi-dirac integral

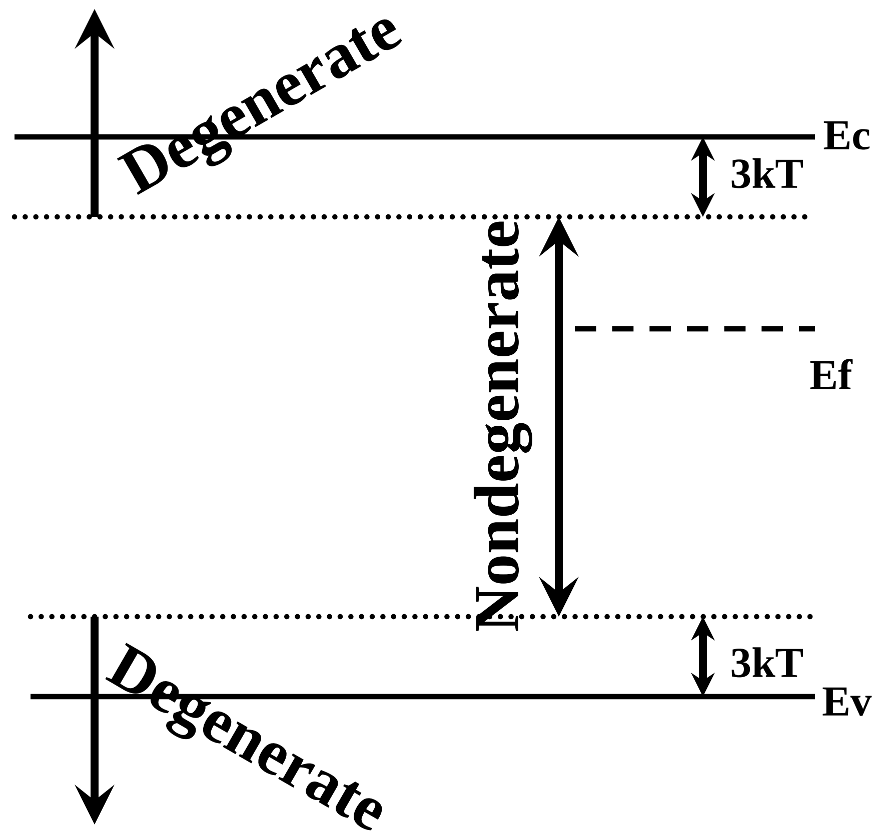
If $E_f < E_c - 3kT$

$$\frac{1}{1+e^{\eta-\eta_c}} \cong e^{-(\eta-\eta_c)}$$

$$F_{1/2}(\eta_c) = \frac{\sqrt{\pi}}{2} e^{(E_f - E_c)/kT}$$

Similarly when $E_f > E_v + 3kT$

$$F_{1/2}(\eta_v) = \frac{\sqrt{\pi}}{2} e^{(E_v - E_f)/kT}$$



Developing the mathematical model for electrons and holes

Useful approximations to the Fermi-dirac integral:

Nondegenerate Case

$$n = N_c e^{(E_f - E_c)/kT}$$

and

$$p = N_v e^{(E_v - E_f)/kT}$$

Developing the mathematical model for electrons and holes

When $n=n_i$, $E_f=E_i$ (the intrinsic energy), then

$$n_i = N_c e^{(E_i - E_c)/kT} \quad or \quad N_c = n_i e^{(E_c - E_i)/kT}$$

and

$$n_i = N_v e^{(E_v - E_i)/kT} \quad or \quad N_v = n_i e^{(E_i - E_v)/kT}$$

$$n = n_i e^{(E_f - E_i)/kT}$$

and

$$p = n_i e^{(E_i - E_f)/kT}$$

Developing the mathematical model for electrons and holes

Other useful Relationships: n - p product

$$n_i = N_c e^{(E_i - E_c)/kT} \quad and \quad n_i = N_v e^{(E_v - E_i)/kT}$$

$$n_i^{\,2} = N_c N_v e^{-(E_c - E_v)/kT} = N_c N_v e^{-E_G/kT}$$

$$\boxed{n_i = \sqrt{N_c N_v}\, e^{-E_G/2kT}}$$

Developing the mathematical model for electrons and holes

Other useful Relationships: n - p product

$$Since \quad n = n_i e^{(E_f - E_i)/kT} \quad and \quad p = n_i e^{(E_i - E_f)/kT}$$

$$np = n_i^2$$

Known as the ***Law of mass Action***

Developing the mathematical model for electrons and holes

Charge Neutrality

- If excess charge existed within the semiconductor, random motion of charge would imply net (AC) current flow. ===> Not possible!

- Thus, all charges within the semiconductor must cancel.

$$q\left[\left(p - N_A^-\right) + \left(N_d^+ - n\right)\right] = 0$$

Mobile + charge
Immobile - charge
Immobile + charge
Mobile - charge

Developing the mathematical model for electrons and holes

Charge Neutrality: Total Ionization Case

N^-_A = Concentration of "ionized" acceptors ~ = N_A

N^+_D = Concentration of "ionized" donors ~ = N_D

$$\left(p - N_A\right) + \left(N_D - n\right) = 0$$

Developing the mathematical model for electrons and holes

Charge Neutrality: Total Ionization Case

$$(p - N_A) + (N_D - n) = 0$$

$$\left(\frac{n_i^2}{n} - N_A\right) + (N_D - n) = 0$$

$$n^2 - n(N_D - N_A) - n_i^2 = 0$$

Watch out for round off error here!

(Use the form that has a positive first term combined with the law of mass action)

$$n = \frac{N_D - N_A}{2} + \sqrt{\left(\frac{N_D - N_A}{2}\right)^2 + n_i^2} \quad or \quad p = \frac{N_A - N_D}{2} + \sqrt{\left(\frac{N_A - N_D}{2}\right)^2 + n_i^2}$$

and

$$pn = n_i^2$$

Developing the mathematical model for electrons and holes

If $N_D >> N_A$ and $N_D >> n_i$

$$n \cong N_D \quad and \quad p \cong \frac{n_i^2}{N_D}$$

If $N_A >> N_D$ and $N_A >> n_i$

$$p \cong N_A \quad and \quad n \cong \frac{n_i^2}{N_A}$$

Developing the mathematical model for electrons and holes

Example:

An intrinsic Silicon wafer has 1e10 cm^{-3} holes. When 1e18 cm^{-3} donors are added, what is the new hole concentration?

$$n \cong N_D = 10^{18}\ cm^{-3}$$

$$p = \frac{n_i^2}{n} = \frac{\left(10^{10}\right)^2}{10^{18}}\ cm^{-3} = 100\ cm^{-3}$$

Developing the mathematical model for electrons and holes

Example:

An intrinsic Silicon wafer has 1e10 cm^{-3} holes. When 1e18 cm^{-3} acceptors and 8e17 cm^{-3} donors are added, what is the new hole concentration?

$$p = \frac{1x10^{18} - 8x10^{17}}{2} + \sqrt{\left(\frac{1x10^{18} - 8x10^{17}}{2}\right)^2 + \left(1x10^{10}\right)^2}$$

$$p = 2x10^{17}\ cm^{-3} = N_A - N_D$$

Developing the mathematical model for electrons and holes

Example:

An intrinsic Silicon wafer at 470K has 1e14 cm^{-3} holes. When 1e14 cm^{-3} acceptors are added, what is the new electron and hole concentrations?

$N_D=0$

$$p = \frac{1x10^{14}}{2} + \sqrt{\left(\frac{1x10^{14}}{2}\right)^2 + \left(1x10^{14}\right)^2}$$

$$p = 1.62x10^{14}\ cm^{-3} \neq N_A - N_D$$

$$n = \frac{\left(1x10^{14}\right)^2}{1.62x10^{14}} = 6.2x10^{13}\, cm^{-3}$$

Developing the mathematical model for electrons and holes

Example:

An intrinsic Silicon wafer at 600K has 4e15 cm^{-3} holes. When 1e14 cm^{-3} acceptors are added, what is the new electron and hole concentrations?

N_D=0

$$p = \frac{1x10^{14}}{2} + \sqrt{\left(\frac{1x10^{14}}{2}\right)^2 + \left(4x10^{15}\right)^2}$$

$$p = 4x10^{15}\ cm^{-3} = n_i \neq N_A - N_D$$

$$n = \frac{\left(4x10^{15}\right)^2}{4x10^{15}} = 4x10^{15}\, cm^{-3} = n_i$$

$$\Downarrow$$

$$Intrinsic\ Material\ at\ High\ Temperature$$

Developing the mathematical model for electrons and holes

Temperature behavior of Doped Material

N-type

QuickTime Movie

P-type

QuickTime Movie